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ABSTRACT

This document was prepared to assist teachers in Virginia in three ways: (1) by assessing student achievement on the competencies adopted by the State Board of Education; (2) by diagnosing student errors in mathematics; and (3) by prescribing instructional activities designed to promote student mastery of skills assessed in the competency program. The materials should prove helpful in strengthening the instructional program and in developing high quality criterion-referenced tests. Following a list of some steps which should be considered in planning a minimum competency program, the first section considers ideas on test construction and assessment. In addition to some test-construction suggestions, sample test items and suggestions for generating questions and answer choices are given by performance indicator. The second section considers instructional suggestions and activities for the skill associated with each performance indicator. Common errors are listed, but the majority of the material presents activities which the classroom teacher would find useful in teaching or reteaching the skill. Appendices present the state adopted competencies for mathematics, vocabulary, and a brief description of the Graduation Competency Test in mathematics. (MNS)

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# minimum competencies

# mathematics

## suggestions for instruction and assessment

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Department of Education
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# PREFACE

The 1978 General Assembly, through legislative action, mandated statewide competencies testing in the areas of reading and mathematics. Passing a competency test in mathematics is a prerequisite for graduation, beginning with the graduating class of 1981. Thus, it is important that local school divisions reevaluate their existing mathematics program in light of this additional graduation requirement.

This publication has been prepared by the Mathematics Service of the Division of Sciences and Elementary Administration to assist teachers in three ways: 1) assessing student achievement on the competencies adopted by the State Board of Education; 2) diagnosing student errors in mathematics and; 3) prescribing instructional activities designed to promote student mastery of skills being assessed in the competency program. Suggestions are separated into two major areas: "Ideas On Test Construction and Assessment" and "Instructional Suggestions And Activities".

Although this publication is designed primarily for teachers of mathematics, it should be noted that the material presented herein may be useful in the instructional programs for handicapped students. It is probably desirable, therefore, that teachers of special education be made aware of the content included in this publication.

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Division of Sciences and
Elementary Education

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# INTRODUCTION

Each of the fifteen minimum competencies in mathematics adopted by the State Board of Education describes definite skills or understandings to be mastered by students. Yet one can safely say that the competency* statements are general in nature. On the other hand, the performance indicators which accompany the competency statements are more specific in terms of intentions and content. The specificity included in the performance indicators is intended to bring to focus the instructional tasks associated with minimum competencies as well as to provide a basis for assessment.

Most educators will agree that the instructional task is clear. Surely there is a need to strengthen existing mathematics programs as well as organize new instructional programs to help students extend their basic computational skills and to develop greater competencies in using those skills.

Even though this publication cannot adequately address all the concerns related to minimum competencies, it is designed to provide teachers with a set of materials which should prove helpful in a) strengthening the instructional program and b) developing high quality criterion-referenced tests. The instructional section includes a listing of errors commonly made by students in the mathematical content defined by the minimum competencies. The section on test construction consists of a set of test specifications and brief guidelines for constructing multiple choice test items.

Whenever a "minimum" is set in education, there is always a danger that the minimum standard will become the "maximum" in due time. Educators

in Virginia are encouraged to keep the minimum competency program in proper perspective. It should help strengthen your existing program by placing emphasis on certain areas of the curriculum, but at the same time, it should not limit the broad, comprehensive programs that now exist in the Commonwealth.

*For a complete listing of the State adopted minimum competencies and performance indicators, see, Appendix A.

## ORGANIZING A MINIMUM COMPETENCY PROGRAM

Listed below are some steps which should be considered in planning the instructional aspect of the Minimum Competencies Program.

1. Study the existing curriculum to determine where the competencies are presently being taught.

2. Add to the curriculum those competencies which are not presently included. Also, adjust curriculum to accommodate those competencies which are now being introduced after the planned administration date for the state test.

3. Acquaint students with the list of competencies, making sure that they understand the skills required for mastery.

4. Identify the responsibilities for teaching the competencies. In other words, be certain tnat each teacher understands his/her responsibility in the instructional program.

5. Identify appropriate instructional materials, activities and strategies.

6. Construct your own competency test to measure the competencies and skills being assessed by the state adopted test. (See the section Ideas on Test Construction and Assessment.)

7. Administer your competency test or portions thereof, at several points, beginning in the eighth grade to assess students' progress in mastering the competencies.

8. Develop a plan for using the results from the state's minimum competency test in providing instruction which will aid students in overcoming weaknesses in mathematics. Schools may wish to use information from this publication to develop diagnostic tests for use with those few students who may fail to master the competencies.

# IDEAS ON TEST CONSTRUCTION AND ASSESSMENT

## INTRODUCTION

Testing is an integral part of an instructional program and with the increased emphasis on statewide assessment in mathematics, the need to develop skills in constructing criterion-referenced tests is paramount. This section, therefore, is intended to give suggestions for developing tests that will consistently measure the same skills and concepts in mathematics being measured in the minimum competency program.

The bulk of this section consists of test specifications which constitute a set of rules to follow in developing test items for a criterion-referenced measure. There are two main functions that a set of criterion-referenced test specifications can fulfill. First, they can communicate to educators the nature of the behavior being measured by the test; and, secondly they provide a set of guidelines so that persons constructing tests can generate a pool of test items to measure specific behaviors. Since the set of specifications in this publication is intended to assist teachers to interpret student performance as it relates to specific competencies, the specifications will be written at a level of technical precision to accommodate this intent.

## COMPONENTS OF TEST SPECIFICATIONS

The test specifications which follow consist of four components. Each component is discussed briefly below.

Performance Indicator - This statement provides a concise overview of the competency being measured.

Sample Test Item - A sample test item is presented. It is typical of the kind that may be generated if the rules set forth in the remainder of the test specifications are followed.

Suggestions for Generating Questions - This section sets forth some rules governing the construction of test items, especially those rules concerning content and form which guide the creation of the test item.

Suggestions for Answer Choices - This section stipulates the rules to be followed in creating both right and wrong choices. In other words, an attempt is made to delineate the kinds of response options that should be included in multiple-choice test items.

CONSTRUCTING A COMPETENCY TEST

With the item specifications developed herein, the remarks which follow are directed towards giving you some guidelines for constructing your own competency test in mathematics. Before you start, be encouraged by the fact that test item construction is an art. While there is no schematic means or prescription for initially achieving a set of consistent and valid items, one can evaluate the test after it has been taken by the students and make judgments about the test.

Your efforts will be strengthened, however, if you pay extremely close attention to the specifications for each performance indicator in constructing your items.

Questions which appear on the State Competency Test will be multiple-choice items. A multiple-choice item consists of a stem which is a question or an incomplete sentence presenting a problem situation, followed by several alternatives, which are possible solutions to the problem. One of the alternatives is the correct answer; the others are plausible

answers, called distractors because their function is to distract students who are uncertain of the correct answer. The stem may also be a graph, or diagram followed by the alternatives relating to it.

Now, as a practical suggestion, read over the guidelines below. Then, begin to write items for the performance indicators. After you have written an item, determine if it satisfactorily meets both the test specifications for the performance indicators and most of the following guidelines.

1. Make directions explicit, so that the student knows exactly what type of response is required.

2. The stem should present a single problem to be solved, expressed clearly and without ambiguity. State the questions so there can be only one interpretation.

3. Make each question independent of other questions. Students are often able to select the correct answer to one item because of information gleaned from another item.

4. Make alternative choices as brief as possible.

5. State the stem in positive form whenever possible. When negative wording is used, emphasize it by underlining or by capitalizing.

6. The best alternative choices to the correct answer are those using common misconceptions or errors commonly made by the students. Excellent distractors can be obtained from incorrect student responses. Please realize that the distractors must be real. The fact that you use three or four distractors does little for either diagnosis or avoiding correct guessing if the distractors do not actually distract.

7. Use the same number of distractors for each item on a test.

8. Try to make all incorrect responses equally plausible or "attractive" to the student.

9. Do not consistently make the correct response longer or shorter than the distractors.

10. Avoid the use of qualifying words such as "always", "never", or "all" as much as possible.

11. Avoid use of the distractors "all of the above" and "none of the above".

12. Avoid using a pattern for the position of the correct response. Many teachers fail to use a, d, and e, as often as they use b and c as distractors.

13. Test items should present a problem worthy of a solution; on the other hand however, test items should be selected that will allow you to measure the skill with which you are concerned. For example, a word problem requiring complicated computation serves little or no purpose if the intent is to diagnose problem solving skills.

14. Use an efficient item format.

    a. List alternatives on separate lines, one under the other, making them easy to read and compare.
    b. Use letters in front of alternatives, to avoid confusion with numerical answers.

Once you have constructed the test, and before administering it to students, actually take the test yourself and, if possible, have some of your colleagues - teachers of subjects other than mathematics - as well as a mathematics teacher, take the test. Invite them to offer constructive criticism.

PERFORMANCE INDICATOR 1.2

Given numerals naming whole numbers less than ten thousand, the student will write the word name for the number.

Sample Test Item:

The word name for 4,026 is

a) four thousand, twenty-six

b) four million, twenty-six

c) forty thousand, twenty-six

d) four hundred, twenty-six

Suggestions for Generating Questions

1. The format for the items will be: "The word name for _____ is".
2. The student will be presented with the numerical form of numbers.

Suggestions for Answer Choices

1. Each item will have four choices, only one will be correct.
2. Distractors will be taken from common errors in writing word names for numbers, such as the incorrect use of place value names and insertion of the word "and".

PERFORMANCE INDICATOR 2.1

Given a set of whole numbers, named by numerals of not more than four digits, the student will arrange the numerals in order from smallest numerical value to largest.

Sample Test Item:

Which set of numbers is arranged in order from smallest to largest?

a) {352, 446, 482, 488}

b) {352, 446, 488, 482}

c) {352, 482, 446, 488}

d) {446, 482, 488, 352}

Suggestions for Generating Questions

1. The format of all items will be: "Which set of numbers is arranged in order from smallest to largest?".
2. The comparison will be between 3 or 4 whole numbers.
3. Numerals in the test items will contain 2, 3, or 4 digits.
4. All numerals in a test item will contain the same number of digits.
5. All test items will have at least one pair of numbers having the same highest place value digit in that item.
6. All test items will have at least one pair of numbers having the same digits in the two highest place values in that item.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.
2. One distractor will be the correct response with the pair of numbers with the same two highest place value digits interchanged.
3. Other distractors should reflect a random interchanging of the numerals.

PERFORMANCE INDICATOR 3.1

Given two, three, or four whole numbers named by numerals of not more than four digits, the student will find the sum.

Sample Test Item:

Add:
5342
608
562
+ 26

a) 6,528

b) 6,438

c) 6,718

d) 6,538

Suggestions for Generating Questions

1. The word "Add" will precede each test item.
2. The items used will contain two, three, or four whole numbers; each numeral containing two, three, or four digits written with corresponding place values in the same column.
3. Write problems in vertical addition format and include the "+" sign.
4. Some problems will require regrouping in one column; others will require multiple regrouping.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.
2. The distractors will be as follows:
   a) failing to regroup when necessary
   b) regrouping when not necessary
   c) carrying the incorrect digit
   d) using incorrect addition fact

PERFORMANCE INDICATOR 3.2

Given two whole numbers named by numerals of not more than three digits, the student will find the difference.

Sample Test Item:

Subtract: 860
- 246

a) 624

b) 626

c) 614

d) 514

Suggestions for Generating Questions

1. The word "Subtract" will precede each test item.
2. The minuend will contain three digits.
3. The subtrahend should contain three digits in at least 60% of the items.
4. Some items will contain a zero in the minuend but not all items will.
5. Some items will contain at least one regrouping; a few should contain two regroupings.
6. The items will be written in vertical form and the subtrahend will be preceded by a "-" sign.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.
2. One distractor will be subtracting the smaller digits in the minuend from digits in the subtrahend (option "b" above).
3. One distractor will be failing to regroup when necessary.
4. One distractor will be regrouping when not necessary.
5. Any remaining distractors will reflect number fact errors.

PERFORMANCE INDICATOR 3.3

Given two whole numbers named by numerals of not more than three digits, the student will find the product.

Sample Test Item:

Multiply: 576
x 32

a) 18422

b) 2880

c) 19432

d) 18432

Suggestions for Generating Questions

1. The word "Multiply" will precede each test item.
2. Test items will be written in vertical form.
3. The multiplicand will contain two or three digits.
4. The multiplier will contain one or two digits.
5. The multiplier will be preceded by an "x" sign.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.
2. Distractors will be one of the following common mistakes:
   a) regrouping error - when a zero is in the multiplicand
   b) will be failing to carry when necessary
   c) improper alignment of partial product, especially when a zero is one of the digits in the multiplier
   d) number fact error in multiplication
   e) adding the digit being "carried" and then multiplying (EX: 28 x 3 gives the incorrect result of 124)

PERFORMANCE INDICATOR 3.4

Given a dividend named by a numeral of no more than four digits and a divisor named by a numeral of no more than two digits, the student will find the quotient.

Sample Test Item:

Divide: $18\overline{)9162}$

a) 59

b) 509

c) 53

d) 564

Suggestions for Generating Questions

1. The word "Divide" will precede each test item.
2. The problem will be written using " $)\overline{\quad}$ " symbol.
3. The divisor will contain one or two digits.
4. The dividend will contain two, three or four digits.
5. No more than 10% of the test items in the set will require a quotient with a zero place holder.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.
2. One distractor will be neglecting to put a zero in the quotient as a place holder.
3. One distractor will be neglecting to use the remainder from the previous step as part of the partial dividend.
4. One distractor will be neglecting to bring down the appropriate digit in the dividend.

5. One distractor will be committing a subtraction or multiplication error as described under performance indicators 3.1 and 3.2.

PERFORMANCE INDICATOR 4.1

Given two decimal fractions named by numerals of no more than four digits and having no more than three places to the right of the decimal point, the student will find the sum or difference.

Sample Test Items:

Subtract:
$$\begin{array}{r} 15.6 \\ -\ 1.47 \\ \hline \end{array}$$

a) 1.413

b) .09

c) 14.13

d) 14.27

Add: 12.7 + 0.36 = _____

a) .487

b) 1.63

c) 13.06

d) 12.43

Suggestions for Generating Questions

1. The word "Add" or "Subtract" will precede each test item.
2. The test items will be evenly distributed with respect to addition and subtraction.
3. Addition test items and subtraction test items will be written in both vertical form and horizontal form.
4. In all subtraction problems the minuend will be greater than the subtrahend.
5. Approximately 50% of the test items in a set for addition, the addends will have the same number of places to the right of the decimal point.
6. Approximately 50% of the test items in a set for subtraction, both the minuend and subtrahend will have the same number of places to the right of the decimal point.
7. There will be at least one test item in subtraction when the minuend is to tenths and the subtrahend to hundredths.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.

2. One distractor will be the result of using the proper procedure for placing the decimal in a multiplication problem. (Use correct digits in answer.)

3. If the test item is in horizontal form, one distractor will reflect an error in alignment of the decimal point.

4. In a subtraction test item of the form described in #7 on the preceding page, one incorrect response will reflect using the hundredths digit of the subtrahend as the hundredths digit of the difference. (Choice "d" in subtraction test item).

PERFORMANCE INDICATOR 4.2

Given two decimal fractions, named by numerals of three digits with the decimal point in any position, and the digits of their product, the student will properly place the missing decimal point in the product.

Sample Test Item:

Place the decimal point correctly in the product.

1.57 x 4.03 = _____

a) 0.63271

b) 632.71

c) 6.3271

c) 63.271

Suggestions for Generating Questions

1. "Place the decimal point correctly in the product" will precede each test item.

2. The test items in the set will be written in horizontal form or vertical form.

3. The multiplicand and multiplier will be decimal fractions of 2 or 3 digits.

4. The position of the decimal point in the factors will vary.

Suggestions for Answer Choices

1. There will be four choices for each test item; only one will be correct.

2. All distractors will have the correct digits in the correct order, but the placement of the decimal point will vary.

PERFORMANCE INDICATOR 4.3

Given a dividend of no more than four digits and no more than three decimal places, a divisor of no more than two digits and no more than one decimal place, and the digits of the quotient, the student will properly place the decimal point in the quotient.

Sample Test Item:

Divide: $1.28 \div 1.6 =$

a) .08

b) .8

c) 8

d) 80

Suggestions for Generating Questions

1. Each test item will be preceded by the word "Divide".

2. Items will be written using both the $)\overline{\quad}$ division symbol and in horizontal format.

3. The divisor and the dividend will be decimal fractions.

4. The position of the decimal point in the dividend and divisor will vary.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.
2. All distractors will have the correct digits in the correct order, but the placement of the decimal point will vary.

PERFORMANCE INDICATOR 5.1

Given two simple fractions which have denominators of 1, 2, 3, 4, or 5, the student will find the product.

Sample Test Item:

Multiply: $\frac{2}{3} \times \frac{1}{5} = \square$

a) $\frac{3}{10}$

b) $\frac{15}{2}$

c) $\frac{2}{15}$

d) $\frac{10}{3}$

Suggestions for Generating Questions

1. The word "Multiply" will precede each test item.
2. Denominators will contain 2, 3, 4, or 5 evenly distributed with appropriate numerators to ensure proper fractions.
3. The test items will be written in horizontal form with "x" sign between the two fractions.
4. Not more than 35% of the test items in the set will require the student to "reduce" the answer to lowest terms.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.
2. The correct answer will be given only in the simplified form.
3. Given items in the form $\frac{a}{b} \times \frac{c}{d}$ the distractor will be as follows:

   a) $\frac{ad}{bc}$

   b) $\frac{bc}{ad}$

   c) $\frac{a + c}{b + d}$

   d) $\frac{a + c}{bd}$

4. One distractor will be the result of finding a common denominator and then multiplying only the numerator.

PERFORMANCE INDICATOR 6.1

Given a percent the student will write the equivalent decimal fraction.

Sample Test Item:

23% = _____

a) .023

b) .23

c) 2.3

d) 23

Suggestions for Generating Questions

1. The format for all items will be: _____% = _____.
2. Numbers used to express percents will be whole numbers between 1 and 100.

3. At least one test item in the set will be a whole number between 1 and 10.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.

2. All distractors will have the correct digits in the correct order, but the decimal point will be placed incorrectly.

PERFORMANCE INDICATOR 7.1

Given a common fraction which has a single digit denominator, the student will write the equivalent decimal fraction.

Sample Test Item:

Express $\frac{4}{5}$ as a decimal fraction.

a) 0.8

b) 4.5

c) 1.25

d) 8

Suggestions for Generating Questions

1. The format of all items will be: "Express _____ as a decimal fraction".

2. Each fraction will be expressed in lowest terms with the numerator less than the denominator (proper fractions).

3. Each fraction will have a single digit denominator.

4. All fractions will be written in the form $\frac{a}{b}$.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.

2. One distractor will be a decimal fraction obtained by using the digits from the numerator and denominator of the fraction. The numerator should be listed first followed by a decimal point.

3. One distractor will be a decimal fraction obtained by dividing the numerator into the denominator.

4. One distractor will be a decimal fraction obtained by using the digits in correct order from the answer but placing the decimal point incorrectly.

5. No more than three digits will be used in any response option.

PERFORMANCE INDICATOR 8.1

Given a number and a percent (from 1-100 inclusive) the student will find the percentage.

Sample Test Item:

15% of 90 is:

a) 13.50

b) 13.5

c) 600

d) 13.65

Suggestions for Generating Questions

1. The format of all items will be: _____% of _____ is:

2. Numbers used to express percents will be whole numbers (from one to one hundred inclusive).

3. The number will be a whole number between 0 and 999.

Suggestions for Answer Choices

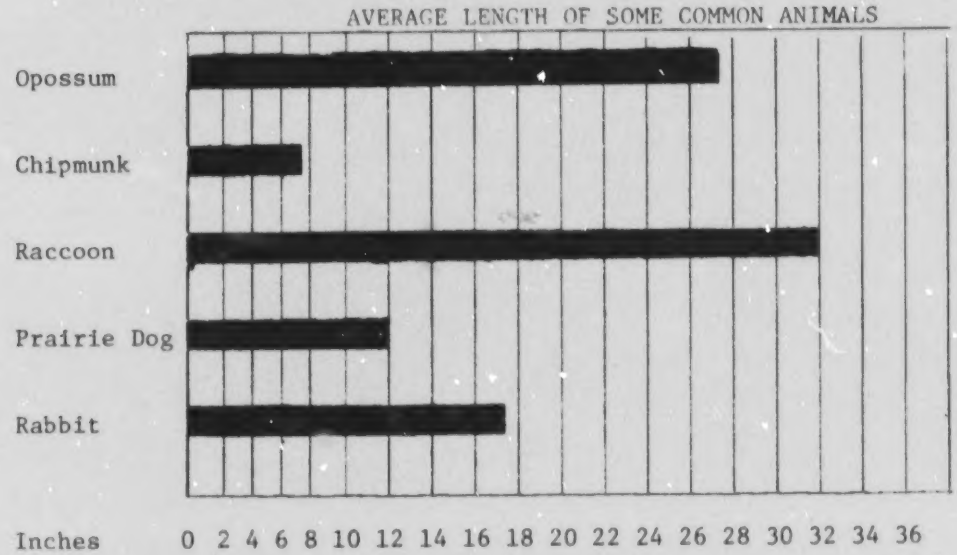
1. There will be four choices for each item; only one will be correct.
2. All response options will be numerical answers.
3. No response will contain more than five digits.
4. The number of digits in the response options should not differ by more than two.
5. Distractors will include:
   a) misplacing the decimal point when changing the percent to a decimal fraction
   b) the results obtained by dividing the number by the percent

PERFORMANCE INDICATOR 9.1

Given a bar or broken-line graph, the student will estimate and compare quantitative information.

Sample Test Item:

Use the graph below to answer questions _____ through _____.


AVERAGE LENGTH OF SOME COMMON ANIMALS
Opossum
Chipmunk
Raccoon
Prairie Dog
Rabbit
Inches
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36

How much longer is the Prairie Dog than the Chipmunk?

a) 4 inches

b) 6 inches

c) 12 inches

d) 5 inches

Suggestions for Generating Questions

1. The student will be given an appropriate graph preceded by the following instructions: "Use the graph below to answer questions _____ through _____".

2. The graph will have an appropriate title.

3. The graph will reflect realistic information pertaining to finance, taxes, population, natural resources, time and temperature.

4. Specifications for broken-line graphs:

   a) The vertical scale will be named and the divisions will be labeled with whole numbers. Horizontal lines should be drawn through these divisions.

   b) The horizontal scale will be named and the divisions labeled.

5. Specifications for bar graphs:

   a) There will be a space shown between the bars of the graph.

   b) Some of the bars should terminate on a horizontal line.

6. Two questions may be written for each graph.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.

2. All distractors will be written in terms of information given on the graph.

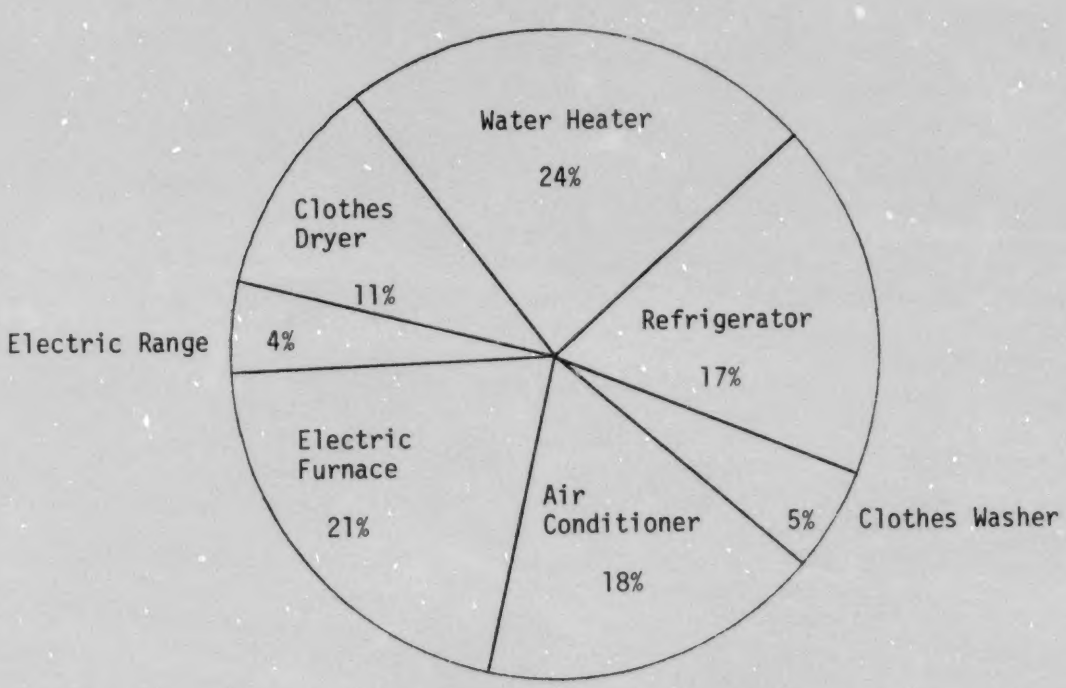
PERFORMANCE INDICATOR 9.2

Given a circle graph in which each sector is labeled as a percent of the whole the student will compare quantitative information.

Sample Test Item:

Use the graph below to answer questions _____ and _____.

Electrical Usage of Major Home Appliances




Which pair of appliances uses the greatest amount of electricity?

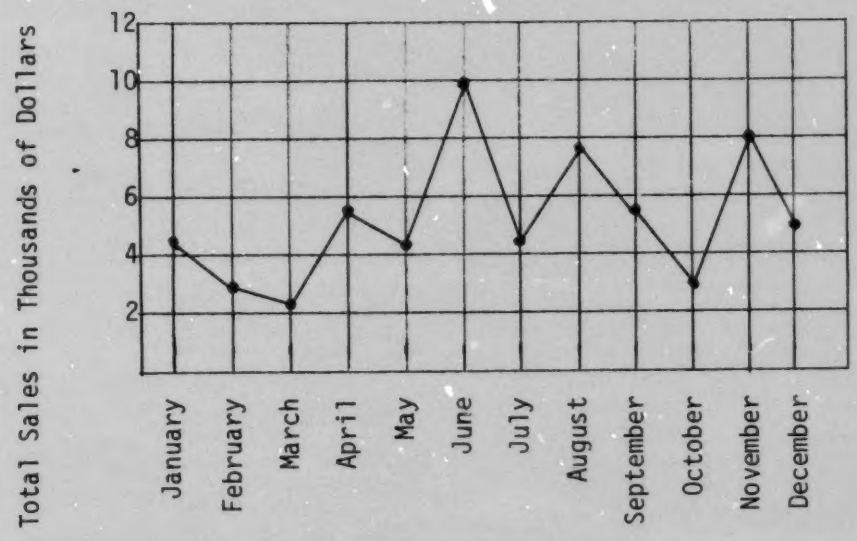
a) air conditioner and furnace

b) refrigerator and air conditioner

c) clothes dryer and water heater

d) furnace and refrigerator

A Company's Monthly Sales for 1977




According to the graph, which of the following statements is true?

a) Total company sales decreased from March to April.

b) Total company sales increased during the period June to July.

c) Total company sales increased from October to November.

d) Total company sales did not change from November to December.

Suggestions for Generating Questions

1. The student will be given an appropriate graph preceded by the statement: "Use the graph below to answer questions _____ through _____."

2. Two questions may be written for each graph. The format of each question will be: "According to the graph, which of the following statements is true?".

3. The graph will have an appropriate title.

4. The content for the graph will include realistic information pertaining to finance, taxes, population and natural resources.

Suggestions for Generating Questions

1. The student will be given a circle graph followed by the statement:

   "Use the graph below to answer questions _____ and _____."

2. The number of sectors in the circle graph will be no more than 7 and no less than 5.

3. Each sector of the circle will include a whole number percent indicating the percent of the circle represented by that sector.

4. Each sector of the circle will include an appropriate label.

5. The graph will have an appropriate title.

6. The graph will reflect realistic information related to budgets, taxes, and natural resources.

7. One item will ask for comparison of the sum of two sectors with one or more sectors. The comparison should involve addition and the concepts of "greatest", "least" or "same as".

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.

2. Distractors will be written as sentence fragments in terms of information given on the graph.

3. Distractors will be drawn from logical misinterpretation of the graph written in terms of information given on the graph.

PERFORMANCE INDICATOR 9.3

Given a bar, broken-line, or circle graph and a statement of inference, the student will state whether the inference is true or false based on information on the graph. (The inference will be a simple statement relative to comparisons of size or frequency and trends of increase, decrease, or constancy.)

Sample Test Item:

Use the graph to answer questions _____ through _____.

5. Specifications for broken-line graphs:

   a) the vertical scale will be named and the divisions labeled with whole numbers

   b) the horizontal scale will be named and the divisions labeled

   c) points of reference will be on a vertical line, a horizontal line, or on the intersection of a vertical and horizontal line (points will be joined by line segments)

6. Bar-graph specifications:

   a) each graph will have a descriptive title and appropriate scale labels

   b) only whole numbers will be used in identifying a scale

   c) there will be a space shown between the bars of the graph

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.

2. The four response options will be simple statements relative to comparisons of size or frequency and trends of increase, decrease, or constancy.

3. Distractors will be statements that contradict information on the graph.

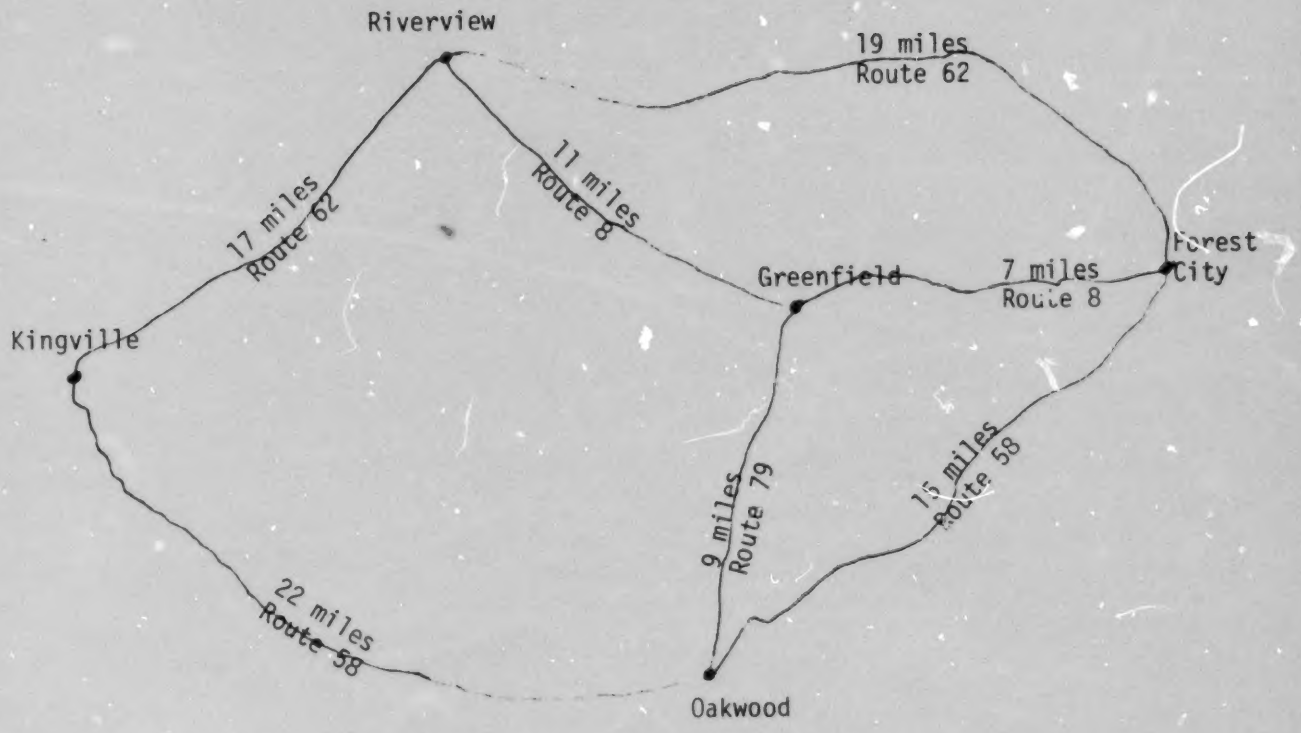
4. The three possible conditions (increase, decrease, constancy) should be distributed in the item set so as not to cue in the correct response.

PERFORMANCE INDICATOR 10.1

Given a map and scale, the student will indicate the route of least mileage between two locations.

Sample Test Items:

Use the map to answer questions 1 and 2.




1. In traveling from Kingville to Forest City, which of the following is the shortest distance?

   a) Route 58 from Kingville to Oakwood, then take route 79 from Oakwood to Greenfield. Finally, take route 8 from Greenfield to Forest City.

   b) Route 62 from Kingville to Riverview, then take route 8 from Riverview to Greenfield. Finally, take route 8 from Greenfield to Forest City.

   c) Route 62 from Kingville to Riverview and then take route 62 from Riverview to Forest City.

   d) Route 58 from Kingville to Oakwood and then take route 58 from Oakwood to Forest City.

2. The distance from Oakwood to Riverview along routes 79 and 8 is:

   a) 90 miles

   b) 87 miles

   c) 20 miles

   d) 17 miles

Suggestions for Generating Questions

1. The students will be presented with a map containing fictitious names for locations followed by the following instructions: "Use the map below to answer questions _____ and _____."

2. The map will include the distance between locations along a specified route or path.

3. The distance between locations will be designated in miles or kilometres.

4. Distance designation will be expressed by a 1 or 2 digit numeral.

5. The number of locations (points) on the map that must be considered in order to obtain a correct response should not exceed four.

6. There will be two items related to each map, one asking for the distance along the shortest route between two locations and the other asking for the distance between two locations along a specific route.

Suggestions for Answer Choices

1. Students will be given four choices; only one will be correct.

2. Distractors for item dealing with "shortest distance" will be choices pertaining to alternate routes.

3. Distractors for items dealing with the distance between two locations along a specified route shall include:

   a) the sum of the route numbers if more than one route is involved

   b) the sum of a route number and the mileage designation

PERFORMANCE INDICATOR 11.1

Given a specified situation and a federal or state income tax table, the student will find the correct amount of tax.

Sample Test Item:

Use the tax table to answer questions 1 and 2.

Partial Tax Table

| Taxable Income is-- | | The Total Number of Exemptions | | |
|---|---|---|---|---|
| Over | But Not Over | 1 | 2 | 3 |
| | | Your tax is-- | | |
| 5,800 | 5,850 | 419 | 264 | 100 |
| 5,850 | 5,900 | 427 | 273 | 108 |
| 5,900 | 5,950 | 436 | 283 | 116 |
| 5,950 | 6,000 | 444 | 292 | 124 |
| 6,000 | 6,050 | 453 | 302 | 133 |
| 6,050 | 6,100 | 461 | 311 | 141 |
| 6,100 | 6,150 | 470 | 321 | 150 |
| 6,150 | 6,200 | 478 | 330 | 158 |
| 6,200 | 6,250 | 487 | 340 | 167 |
| 6,250 | 6,300 | 495 | 349 | 175 |
| 6,300 | 6,350 | 504 | 359 | 184 |
| 6,350 | 6,400 | 512 | 368 | 192 |
| 6,400 | 6,450 | 521 | 378 | 201 |
| 6,450 | 6,500 | 529 | 387 | 210 |
| 6,500 | 6,550 | 538 | 397 | 219 |
| 6,550 | 6,600 | 546 | 406 | 229 |
| 8,400 | 8,450 | 890 | 748 | 580 |
| 8,450 | 8,500 | 900 | 757 | 590 |
| 8,500 | 8,550 | 909 | 767 | 601 |
| 8,550 | 8,600 | 919 | 776 | 611 |
| 8,600 | 8,650 | 928 | 786 | 622 |
| 8,650 | 8,700 | 938 | 795 | 632 |
| 8,700 | 8,750 | 947 | 805 | 643 |
| 8,750 | 8,800 | 957 | 814 | 653 |

1. Find the amount of tax a single person would pay on $8,625 of taxable income, if the total number of exemptions claimed is 2.

   a) $928

   b) $786

   c) $776

   d) $632

2. Find the amount of tax you would pay on $5,950 of taxable income if you were single and claimed only yourself as a dependent.

   a) $436

   b) $444

   c) $292

   d) $283

Suggestions for Generating Questions

1. The students will be given a federal income tax table preceded by the following instructions: "Use the tax table to answer questions _____ through _____."

2. Questions will be consistent with the tax table used.

3. Each item will ask for the amount of tax on a specified amount of taxable income in relationship to a specified situation (filing status and exemptions).

4. Incomes will be expressed in whole numbers of dollars.

Suggestions for Answer Choices

1. The student will be given four choices; only one will be correct.

2. Response options will be written as whole numbers of dollars.

3. All distractors will be numbers from the tax table taken from rows and columns near, location wise, to the correct answer.

4. One distractor will be from the same column as the correct response.

5. One distractor will be from the same row adjacent to the correct response.

6. The other distractor will be from another column.

7. The order of correct responses will be random.

PERFORMANCE INDICATOR 11.2

Given the cost of an item(s) and a sales tax table, the student will find the correct amount of sales tax.

Sample Test Item:

Use the sales tax table to answer the following questions.

TAX TABLE

4% Virginia Sales Tax Table

| Amount of Sale | Tax | Amount of Sale | Tax | Amount of Sale | Tax |
|---|---|---|---|---|---|
| .01 — .14 | .00 | 14.88 — 15.12 | .60 | 29.88 — 30.12 | 1.20 |
| .15 — .34 | .01 | 15.13 — 15.37 | .61 | 30.13 — 30.37 | 1.21 |
| .35 — .59 | .02 | 15.38 — 15.62 | .62 | 30.38 — 30.62 | 1.22 |
| .60 — .84 | .03 | 15.63 — 15.87 | .63 | 30.63 — 30.87 | 1.23 |
| .85 — 1.14 | .04 | 15.88 — 16.12 | .64 | 30.88 — 31.12 | 1.24 |
| 1.15 — 1.34 | .05 | 16.13 — 16.37 | .65 | 31.13 — 31.37 | 1.25 |
| 1.35 — 1.59 | .06 | 16.38 — 16.62 | .66 | 31.38 — 31.62 | 1.26 |
| 1.60 — 1.84 | .07 | 16.63 — 16.87 | .67 | 31.63 — 31.87 | 1.27 |
| 1.85 — 2.14 | .08 | 16.88 — 17.12 | .68 | 31.88 — 32.12 | 1.28 |
| 2.15 — 2.34 | .09 | 17.13 — 17.37 | .69 | 32.13 — 32.37 | 1.29 |
| 2.35 — 2.59 | .10 | 17.38 — 17.62 | .70 | 32.38 — 32.62 | 1.30 |
| 2.60 — 2.84 | .11 | 17.63 — 17.87 | .71 | 32.63 — 32.87 | 1.31 |
| 2.85 — 3.14 | .12 | 17.88 — 18.12 | .72 | 32.88 — 33.12 | 1.32 |
| 3.15 — 3.34 | .13 | 18.13 — 18.37 | .73 | 33.13 — 33.37 | 1.33 |
| 3.35 — 3.59 | .14 | 18.38 — 18.62 | .74 | 33.38 — 33.62 | 1.34 |
| 3.60 — 3.84 | .15 | 18.63 — 18.87 | .75 | 33.63 — 33.87 | 1.35 |
| 3.85 — 4.14 | .16 | 18.88 — 19.12 | .76 | 33.88 — 34.12 | 1.36 |
| 4.15 — 4.34 | .17 | 19.13 — 19.37 | .77 | 34.13 — 34.37 | 1.37 |
| 4.35 — 4.59 | .18 | 19.38 — 19.62 | .78 | 34.38 — 34.62 | 1.38 |
| 4.60 — 4.84 | .19 | 19.63 — 19.87 | .79 | 34.63 — 34.87 | 1.39 |

An item costs $1.40. How much sales tax should be charged?

a) $0.67

b) $ .05

c) $ .06

d) $ .07

Suggestions for Generating Questions

1. The students will be given a Sales Tax Table preceded by: "Use the sales tax table to answer questions _____ and _____."
2. Questions will be consistent with the sales tax table used.
3. The cost of the item will be less than $35.00.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.
2. One distractor will be from the same column one row above the correct response.
3. One distractor will be from the same row in a column adjacent to the correct response.
4. One distractor will be in the same column in the row below the correct response.
5. The order of correct responses will be a random choice.

PERFORMANCE INDICATOR 12.1

Given a set of figures, the student will identify those which best represent the concept of parallelism.

Sample Test Item:

Which two lines are parallel?

a)

b)

c)

d)

Suggestions for Generating Questions

1. Students will be given figures representing the concept of parallelism preceded by an appropriate statement.

2. The items will require students to identify one of the following: pairs of parallel lines or other figures or objects representing the concept of parallelism.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.

2. Distractors:

   a) intersecting lines

   b) line intersecting two segments (not parallel)

   c) lines which if continued would intersect

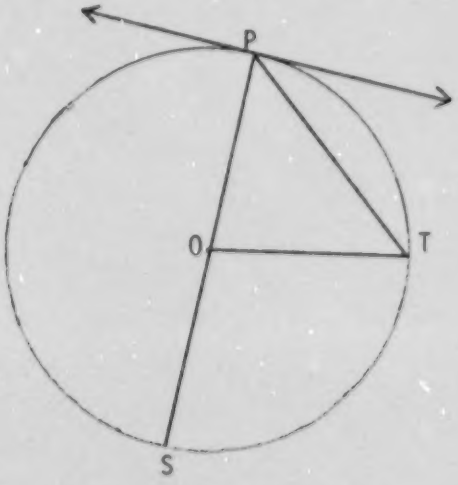
3. Use correct drawings of lines including arrows, but do not name any points.

4. Other distractors should be in keeping with picture or diagram used.

PERFORMANCE INDICATOR 12.2

Given an appropriate drawing of a circle, the student will identify the center, a radius, and a diameter.

Sample Test Item:



Which line segment is a diameter of the circle?

a) $\overline{PT}$

b) $\overline{OS}$

c) $\overline{PS}$

d) $\overline{PO}$

Suggestions for Generating Questions

1. The student will be given an appropriate figure.
2. The circle will be labeled with no more than one diameter, one chord (other than diameter), three radii, and a center. One tangent as one additional point on the circle may be included.
3. The circle will have a diameter of approximately 5 cm.
4. The student will be asked:
   a) Which line segment is a diameter?
   b) Which line segment is a radius?
   c) Which point names the center of the circle?

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.
2. Distractors for diameters will be:
   a) another chord
   b) a radius
   d) two points
3. Distractors for radii will be:
   a) diameter
   b) chord
   c) two points not connected by a line segment
4. Distractors for center will be other points associated with the circle.

PERFORMANCE INDICATORS 13.1 and 13.2

Given the dimensions of a rectangular region, the student will find the area of the region.

Given the lengths of the sides of a rectangular region, the student will find the perimeter of the region.

Sample Test Items:

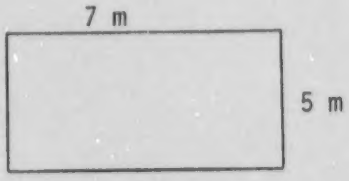
What is the area of this figure?

a) 70 $m^2$

b) 12 $m^2$

c) 24 $m^2$

d) 35 $m^2$



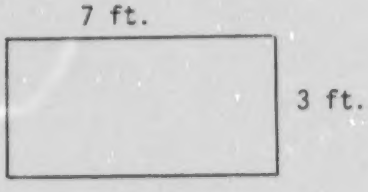

What is the perimeter of this figure?

a) 21 feet

b) 10 feet

c) 20 feet

d) 13 feet




Suggestions for Generating Questions

1. The student will be asked, "What is the area of this figure?" or "What is the perimeter of this figure?".
2. The student will be given an appropriate figure representing a rectangular region.

Suggestions for Answer Choices

1. There will be four choices per item; only one will be correct.
2. All choices will be expressed in correct units.

3. Distractors for area

   a) One distractor will be the sum of the measures of two adjacent sides in square units.

   b) One distractor will be twice the area.

   c) One distractor will be the perimeter expressed in square units.

   d) One distractor will be the sum of two adjacent sides, not expressed in square units.

4. Distractors for perimeter

   a) One distractor will be the product of the measures of two adjacent sides.

   b) One distractor will be the sum of the measures of two adjacent sides.

   c) One distractor will be the sum of the measures of three sides.

PERFORMANCE INDICATOR 14.1

Given a list of units of measure, the student will identify those indicating length (metre, centimetre, kilometre; foot, inch, mile) mass/weight (kilogram, gram; pound) and capacity (litre, millilitre; pint, quart, gallon), or vice versa.

Sample Test Items:

1. The distance you travel when you go to the beach can be measured in:

   a) kilometres

   b) litres

   c) pounds

   d) grams

2. Which measures mass?

   a) metre

   b) kilogram

   c) litre

   d) centimetre

3. Which measures capacity?

   a) grams

   b) metres

   c) litres

   d) degrees

Suggestions for Generating Questions

1. The student will be given a situation involving linear measure such as distance, height, or room dimensions.

2. For mass or weight the student will be given the directions, "Which measures mass or weight?".

3. For capacity the student will be given the directions, "Which measures capacity?".

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.

2. Distractors for items dealing with linear measure will be mass or capacity units.

3. Distractors for items dealing with mass or weight will be linear units or capacity units.

4. Distractors for items dealing with capacity will be linear or mass/ weight units.

PERFORMANCE INDICATOR 14.2

Given a drawing of either a Celsius or Fahrenheit thermometer, the student will write the indicated temperature to the nearest degree.

Sample Test Item:

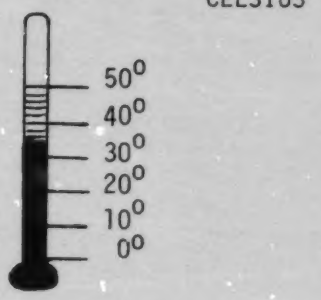
What is the temperature to the nearest whole degree?

a) $31^{o}$

b) $30^{o}$

c) $35^{o}$

d) $20^{o}$




Suggestions for Generating Questions

1. Students will be asked, "What is the temperature to the nearest degree?".

2. A thermometer will be pictured either labeled "Celsius" or "Fahrenheit". Degrees will be named in $10^{o}$ intervals with 4 horizontal markings between each $10^{o}$.

3. Some items will have temperature marked on the multiples of $10^{o}$ positions; other items will have temperature marked on the intermediate markings.

4. All temperatures shown will be greater than or equal to $10^{o}$.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.

2. Distractors will be those normally associated with misreading a scale.

3. One distractor will be reading of temperature at $+1^{o}$ or $-1^{o}$ from actual reading.

PERFORMANCE INDICATOR 14.3

Given two times (to the nearest ½-hour designation) within a 12-hour time interval, the student will determine the elapsed time.

Sample Test Item:

A family left Norfolk at 10:00 a.m. and arrived in Raleigh at 2:30 p.m. How long were they traveling?

a) 8 hours and 30 minutes

b) 8 hours

c) 12 hours and 30 minutes

d) 4 hours and 30 minutes

Suggestions for Generating Questions

1. The student will be given a problem of two sentences: a) a statement which states two times of day expressed in the form ____:____ each; (time will be expressed either on the hour or half-hour within a 12 hour period) b) a question, asking the time elapsed between the two times given.

2. Questions should be realistic.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.

2. Distractors will be:

   a) adding hours and minutes

   b) subtracting hours and minutes, always the smaller hour from the larger

   c) subtracting just the hours

PERFORMANCE INDICATOR 15.1

Given an amount of money, $1, $5, or $10, and the total amount of purchase, the student will determine the correct change.

Sample Test Item:

John bought a ball for $2.25 including tax. He gave the clerk a $5 bill. What amount of change should he receive?

a) $3.75

b) $1.75

c) $2.25

d) $2.75

Suggestions for Generating Questions

1. The student will be given:

   a) a purchase price including sales tax

   b) an amount of money

   c) an interrogative, asking for the amount of change from the purchase

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.
2. Distractors will include:

   a) subtraction of purchase price without regrouping

   b) subtraction of dollars with addition of cents

   c) using other common subtraction errors

PERFORMANCE INDICATOR 15.2

Given a specified number of hours worked and an hourly rate of pay, the student will compute the wages.

Sample Test Item:

A worker earns $2.50 per hour. How much will he earn in 40 hours?

a) $ 2.90

b) $100.00

c) $42.50

d) $1000.00

Suggestions for Generating Questions

1. Students will be presented a problem giving earnings per hour not to exceed $5.00 per hour, and the number of hours worked not to exceed 40 hours expressed in whole numbers.

2. The problem will be two or three sentences.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.

2. Distractors will be:

   a) addition of hours and money with disregard of place value

   b) addition of hours and money with place value correct

   d) mistake in multiplication fact

PERFORMANCE INDICATOR 15.3

Given the gross earnings and the amounts of deduction for social security and federal and state taxes, the student will compute the net earnings.

Sample Test Item:

This is a worker's pay stub. What is the take-home pay after deductions?

| | |
|---|---|
| GROSS PAY | $106.00 |
| Deductions: | |
| Federal Tax | $ 21.00 |
| State Tax | $ 8.00 |
| F.I.C.A. | $ 8.00 |

a) $143.00

b) $ 77.00

c) $ 69.00

d) $ 98.00

Suggestions for Generating Questions

1. The student will be given the amount of gross pay and each of the following deductions: federal tax, state tax, and F.I.C.A. The student will be given the directions: "What is the take-home pay after deductions?"

2. Some items will be a representation of actual pay stubs. Other items will be in the form of a word problem.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.

2. Distractors will include:

   a) adding all deductions to gross pay

   b) omission of one deduction in making the subtraction

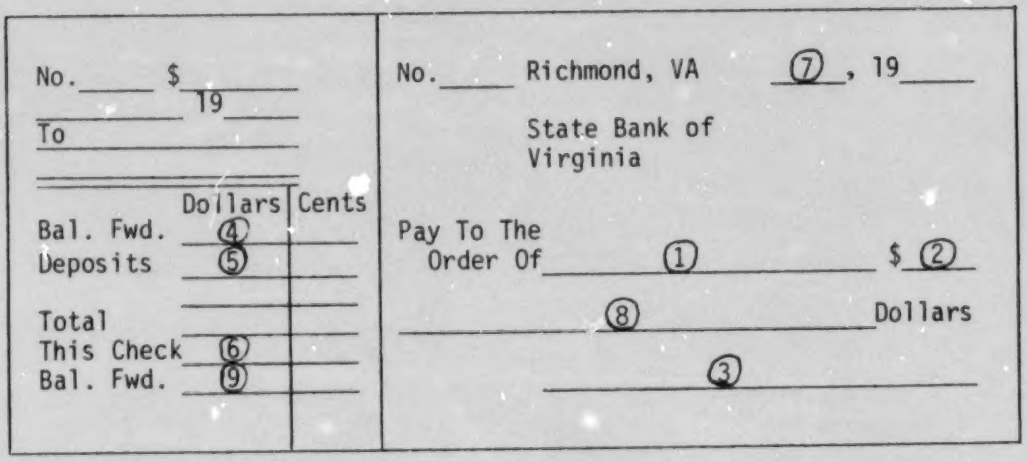
PERFORMANCE INDICATORS 15.4 and 15.5

Given the appropriate information, the student will write a check.

Given the appropriate information, the student will complete a check stub.

Sample Test Item:

Use the check stub and sample check to answer questions 1 through 6.




Information:

On May 10, 1978, Mary Jones wrote a check for $27.50 to pay for a sweater from Sears, Roebuck Company. Before writing this check, Mary had $186.50 in her checking account. Since writing this check, she has deposited $100.00.

1. In the area marked ① what should be written?

   a) Mary Jones

   b) Sears, Roebuck Company

   c) $27.50

   d) $100.00

2. In the area marked ② what should be written?

   a) $186.50

   b) $ 27.50

   c) May 10, 1978

   d) $100.00

3. In the area marked (3) what should be written?

   a) May 10, 1978

   b) Sears, Roebuck Company

   c) Sweater

   d) Mary Jones

4. In the area marked (4) what should be written?

   a) $100.00

   b) $ 27.50

   c) $186.50

   d) May 10, 1978

5. In the area marked (5) what should be written?

   a) $100.00

   b) $ 27.50

   c) $186.50

   d) Mary Jones

6. In the area marked (6) what should be written?

   a) $ 27.50

   b) Sears, Roebuck Company

   c) $100.00

   d) $186.50

Sample Test Item:

Using the check record, what is the new balance after checks #672 and #673 are written?

| | | | | |
|---|---|---|---|---|
| | | | | 120 00 |
| 672 | 8/30 | To: Sears<br>For: radio | 26 00 | |
| 673 | 9/6 | To: Auto Supply Company<br>For: battery | 32 00 | |

a) $ 88.00

b) $ 94.00

c) $ 62.00

d) $178.00

Suggestions for Generating Questions

1. The student will be given information including the date of writing a check, the payee, the payer, the balance before writing the check, the deposit after writing the check, and the amount of the check.
2. The student will be given a diagram of a check and stub.
3. Different forms of check records should be used.

Suggestions for Answer Choices

1. There will be four choices per item; only one will be correct.
2. Distractors will be varied combinations of the information given:

   (1) Payee, payer, amount of check, amount of deposit

   (2) Balance brought forward before the check was written, the date, and the deposit

(3) Payee, item, and payer's name in type

(4) Deposit, amount of check, and the date

(5) Balance brought forward before check was written, the amount of the check, and the payer

(6) The payee, the amount of deposit, and the balance before the check was written

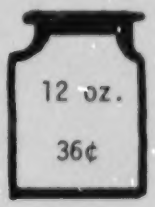
PERFORMANCE INDICATOR 15.6

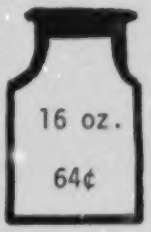
Given the prices of similar packaged goods, the student will determine and compare unit prices.

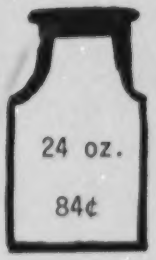
Sample Test Item:

Which bottle of juice is the cheapest per ounce?

a)


b) 


c) 


d) 


Suggestions for Generating Questions

1. The student will be asked, "Which container is the cheapest per unit of measure being used?".
2. Labeled containers will be presented pictorially including the product and units of measure.
3. The containers will be drawn proportionate to size.
4. The distractors are size of containers and varied unit prices.

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.

2. The distractors will be the size of container and varied unit price.

PERFORMANCE INDICATOR 15.7

Given the regular price of an item and a rate of discount, the student will compute the amount of discount.

Sample Test Item:

A desk regularly sells for $125.00. It is advertised at a 20% discount. Find the amount of discount.

a) $25.00

b) $ 2.50

c) $145.00

d) $100.00

Suggestions for Generating Questions

1. The statement will include the regular price of an item and the percent of discount. The student will be asked, "What is the amount of discount?".

Suggestions for Answer Choices

1. There will be four choices for each item; only one will be correct.
2. The distractors will be:
   a) add percent to regular cost of item
   b) misplaced decimal point in answer
   c) subtracting amount of discount from regular price

# INSTRUCTIONAL SUGGESTIONS AND ACTIVITIES

## INTRODUCTION

The information contained in this section is organized around those skills dictated by the state adopted minimum competencies for mathematics. That is, an abbreviated description of the skill associated with the performance indicator is presented, followed by examples of errors commonly made by students with deficiencies in that skill area. The major portion of the section however includes a compilation of suggested instructional activities and strategies. The information included in this section is not intended to be all-inclusive but is designed to provide a starting point in planning appropriate instructional programs.

Initial steps in planning such programs must include careful analysis of the content suggested by the minimum competencies and performance indicators in order to identify the skills and concepts needed for mastery of the competencies by students. Also, consideration must be given to identifying key subskills which are prerequisites to acquiring the terminal skill. Awareness of these components is essential in diagnosing student weaknesses and providing appropriate instruction, both remedial and developmental.

Determining methods appropriate for remedial instruction is not a simple matter. It requires a careful analysis of students' work as well as an analysis of the conceptual development of the content being taught. It will be necessary to observe many samples of students' work, interview students on an individual basis and encourage them to explain their work to you. One must remember that learning and erring are individual behaviors.

Although it may appear on the surface, or after some investigation, that individuals share the same learning or the same erring pattern, it is likely they do so for reasons unique to each of them. If the learning of mathematics by students is to be enhanced, teachers must take the time necessary to diagnose the nature of students' misunderstandings.

The instructional suggestions and activities presented herein focus on both concept development and skill mastery. Many of the suggested procedures and strategies for teaching both concepts and skills involve the use of visual aids and manipulative materials. The drill and practice activities such as games and puzzles focus mainly on skill development. Although worksheets have not been emphasized as a source of drill exercises, this does not mean they may provide practice opportunities for students. However, it is felt that they must not be used to the exclusion of other appropriate techniques.

Although the instructional suggestions and activities do not specifically address reading mathematics, consideration should be given to helping students develop the necessary skills to read with comprehension. This means that classroom instruction must focus on vocabulary development as well as interpretation of symbols. Students should not be asked to give "dictionary" definitions of terms; but they should be able to understand the meanings of terms and symbols in the context of the problems they will be required to solve.

Providing appropriate instruction for students with deficiencies in mathematical skills is a challenging undertaking. Although it is not an easy task, the rewards are great; hence, it is our hope that the following information will assist you in planning such instructional programs.

## 1.2 WRITING WORD NAMES FOR WHOLE NUMBERS

### Common Errors

1. Reading the number incorrectly.

   Example: The student reads 3600 as thirty-six thousand.

2. Determining place value incorrectly.

### Instructional Suggestions and Activities

Although spelling of the word names is a concern, it should not be the predominant focus of instruction. That is, you may wish to provide some opportunity for practice but not devote an inordinate amount of time to it.

Reading and writing the incorrect word name for a number may be the result of confusion about place value. Asking a student to randomly, as well as sequentially, name the place value of each digit of a numeral may help pinpoint the problem

Example: What is the place value of the 5 in 1356, of the 1, of the 6?

Practice in identifying place value may be in the form of writing numerals in an expanded form, completing place value charts, or crossword and cross-number puzzles.

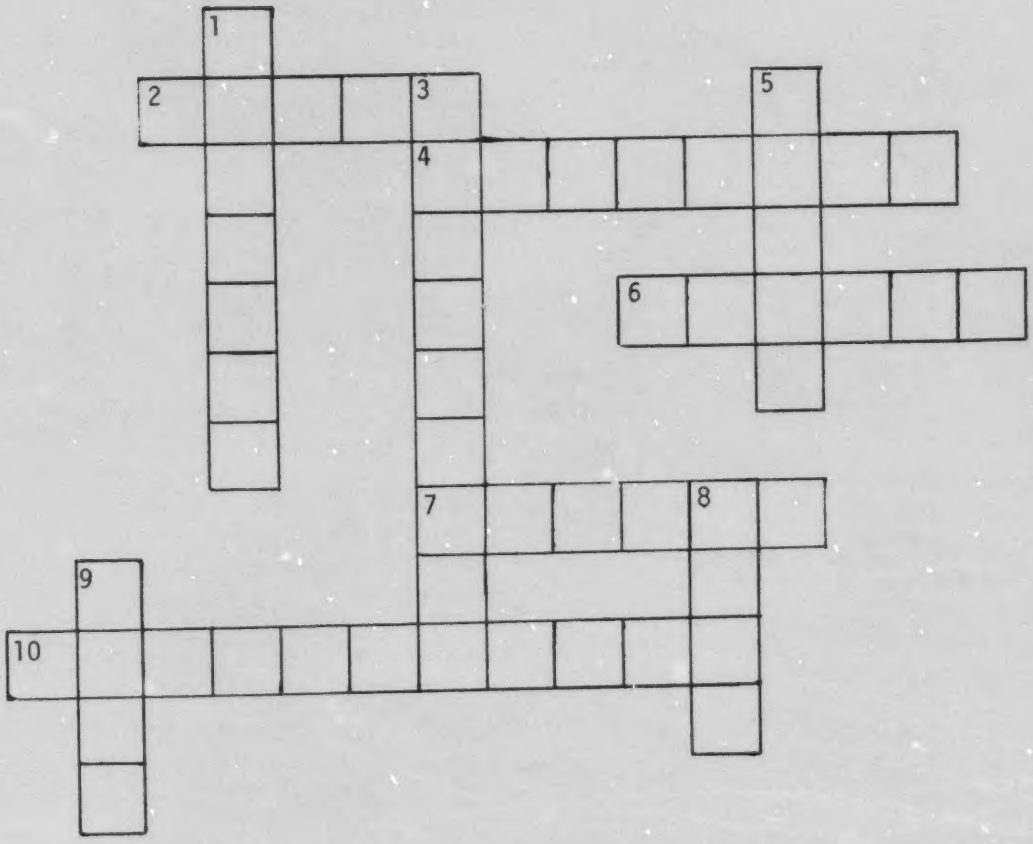
Examples:

$$\begin{aligned} 597 &= 500 + 90 + 7 \\ &= 5 \text{ hundreds} + 9 \text{ tens} + 7 \\ &= \text{five hundred ninety-seven} \end{aligned}$$

Complete the Chart

| Thousands | Hundreds | Tens | Ones | Number |
|---|---|---|---|---|
| | 2 | 7 | 4 | - |
| 1 | 0 | - | 2 | 1052 |
| 3 | - | 6 | - | 3265 |
| | - | - | 9 | 579 |
| - | - | - | - | 8041 |

CROSS-WORD PUZZLE




DOWN

1. Word name for 15
3. Place value of 7 in 7462
5. Word name for digit in tens' place in 275
8. Place value of 5 in 53
9. Word name for 4

ACROSS

2. Word name for 8
4. Place value of 2 in 1263
6. Word name for 20
7. Word name for 90
10. Word name for 47

## 2.1 COMPARING NUMERICAL VALUES

### Common Errors

1. Arranging the numerals in the opposite order of the directions, e.g., arranging them in order from largest numerical value to smallest when asked to arrange them from smallest numerical value to largest.

2. Placing one or more members of the set in the incorrect order.

### Instructional Suggestions and Activities

Students generally do well on items of this nature; however, if students arrange numerals in the opposite order of the directions, it is probably sufficient to remind them to read carefully.

Before being able to order a set of whole numbers, each named by numerals of less than 5 digits, a student must be able to read and write numerals for numbers 0 - 9,999. Comparing numerical values of two whole numbers, 0 - 99 should precede ordering a set of more than two such whole numbers. Then, follow the same sequence with whole numbers 100 - 999. That is, compare the numerical values of two such numbers before ordering a set of more than two.

If students are making random or inconsistent errors, for example, those that may indicate some weakness but not necessarily a lack of understanding of place value, it may be helpful to let them use a number line and encourage them to think of the numbers in terms of which comes first in counting.

If students have difficulty comparing numerical values, it generally occurs when the numerals have the same number of digits or the same digits as well as the same number of digits. For example, ordering 1011, 1110, 1010, 1101 from largest numerical value to smallest. An emphasis on place value and a systematic procedure may be helpful. An example of such an approach is listed below.

Example: Determine which is larger, 1011 or 1010.

(This procedure presupposes that the student can arrange the numbers 0-9 in order from smallest numerical value to largest and vice versa.)

1. Look at the digits in the thousands' place of each numeral. The numeral having the digit of greater numerical value names the larger number. If the digits are the same, as in this case, go to step 2.

2. Look at the digit in the hundreds' place of each numeral. The numeral having the digit of greater numerical value names the larger number. If the digits are the same, as in this case, go to step 3.

3. Where do you suppose we look this time? Right, look at the digit in the tens' place of each numeral. The numeral having the digit of greater value names the larger number. If the digits are the same, as in this case, to to step 4.

4. Look at the digit in the ones' place of each numeral. The numeral having the digit of greater value names the larger number. Therefore, 1011 is larger than 1010.

## 3.1 ADDITION OF WHOLE NUMBERS

### Common Errors

1. Making a number fact error.

Examples:

$$\begin{array}{r} 29 \\ +\ 36 \\ \hline 66 \end{array} \qquad \begin{array}{r} 55 \\ +\ 82 \\ \hline 147 \end{array}$$

2. Improper alignment of the addends.

Example: $5 + 172 + 12 =$ _____

$$\begin{array}{r} 5 \\ 172 \\ +\ 12 \\ \hline 792 \end{array}$$

3. Regrouping error.

Example:

$$\begin{array}{r} 624 \\ +\ 738 \\ \hline 1352 \end{array} \qquad \begin{array}{r} 789 \\ +\ 357 \\ \hline 1046 \end{array} \qquad \begin{array}{r} 659 \\ +\ 152 \\ \hline 901 \end{array}$$

## Instructional Suggestions and Activities

After determining which number facts are causing the student to make errors, it may be necessary to provide practice working with the correct number facts. A few minutes practice each day is generally more effective in helping the student than more infrequent and lengthy practice sessions. Practice may be in the form of cross-number puzzles, flash cards, "beat the clock" or "beat the calculator" games. In "beat the clock" games, a student can compete with himself and try to better his time. (A sand timer works well in class.) In "beat the calculator" games, one student uses paper and pencil or mental arithmetic and another uses a calculator. In either case not all problems should contain multidigit numbers or troublesome facts. Encourage success and confidence as well as practice in the particular number combinations. Combinations frequently causing difficulty are those whose sums are 14, 15, 16, and 17.

Improper alignment of addends as well as regrouping errors may be the result of a lack of understanding of place value. Using place value charts or expanded notation may be helpful in correcting these problems. Several examples follow.

Examples:

$$\begin{array}{rl} 827 = & 800 + 20 + 7 \\ \underline{+\ 79} = & \underline{\qquad 70 + 9} \\ & 800 + 90 + 16 = 800 + 100 + 6 \end{array}$$

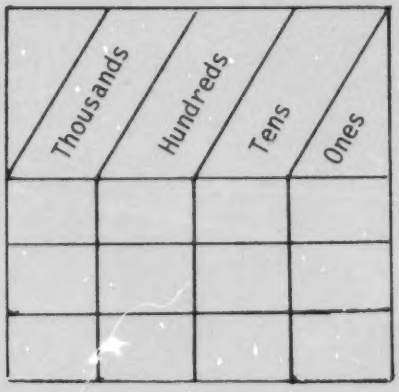
$$= 906$$

$$\begin{array}{rl} 827 = & 8 \text{ hundreds} + 2 \text{ tens} + 7 \\ \underline{+\ 79} = & \underline{\qquad\qquad 7 \text{ tens} + 9} \\ & 8 \text{ hundreds} + 9 \text{ tens} + 16 \end{array}$$

$$= 8 \text{ hundreds} + 10 \text{ tens} + 6$$

$$= 9 \text{ hundreds} + 0 \text{ tens} + 6$$

$$= 906$$



In working with students who are making regrouping errors, it is important that instruction and practice include examples which require regrouping only once as well as those which require regrouping two or three times. Because a student can correctly work a problem which requires regrouping once (for example, regrouping from the ones' place to the tens' place) does not imply that he can correctly work one which requires regrouping more than once (for example, regrouping from the ones' place to the tens' place and from the tens' place to the hundreds' place). Therefore, it will be necessary to view the students' work in both types of problems to determine their particular deficiency.

## 3.2 SUBTRACTION OF WHOLE NUMBERS

### Common Errors

1. Making a number fact error.

   Examples:

$$\begin{array}{r} 79 \\ -\ 26 \\ \hline 52 \end{array} \qquad \begin{array}{r} 125 \\ -\ 39 \\ \hline 87 \end{array}$$

2. Making a reversal error, i.e., subtracting the number of smaller numerical value from the one of larger value, regardless of whether it is in the minuend or subtrahend.

   Examples:

$$\begin{array}{r} 458 \\ -\ 174 \\ \hline 324 \end{array} \qquad \begin{array}{r} 63 \\ -\ 54 \\ \hline 11 \end{array}$$

3. Regrouping incorrectly.

   Examples:

$$\begin{array}{r} 482 \\ -\ 239 \\ \hline 253 \end{array} \qquad \begin{array}{r} 956 \\ -\ 458 \\ \hline 598 \end{array} \qquad \begin{array}{r} 305 \\ -\ 168 \\ \hline 47 \end{array}$$

4. Improperly aligning the minuend and subtrahend.

   Example: 521 - 39 = _____

$$\begin{array}{r} 521 \\ -\ 39 \\ \hline 131 \end{array}$$

Instructional Suggestions and Activities

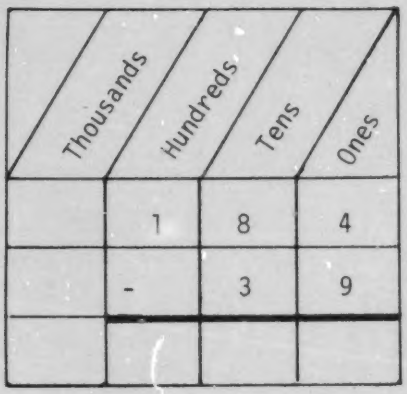
At the high school level, regrouping errors may be the most frequent. Because a student can do a subtraction problem which requires regrouping from the tens' place to the ones' place does not imply he can do one which requires regrouping from the hundreds' place to the tens' place or across intermediate zeros. A conceptual development of subtraction may be structured as follows:

1. Subtract without regrouping.
2. Subtract with regrouping from the tens' place to the ones' place.
3. Subtract with regrouping from the hundreds' place to the tens' place or from the tens' place to the ones' place.
4. Subtract with regrouping from the hundreds' place to the tens' place and from the tens' place to the ones' place.
5. Subtract with regrouping across intermediate zeros.

An understanding of place value is essential to regrouping. Ample opportunity for practice in writing numerals in expanded notation (for example, 329 = 3 x 100 + 2 x 10 + 9) and identifying place value of digits must be provided. Using a place value chart or writing the change in words often helps the student who is having trouble with regrouping.

Examples:

$$\begin{array}{rl} 84 = & 8 \text{ tens and } 4 \text{ ones} = 7 \text{ tens and } 14 \text{ ones} \\ \underline{-\ 39} = & \underline{3 \text{ tens and } 9 \text{ ones}} = \underline{3 \text{ tens and } 9 \text{ ones}} \\ & 4 \text{ tens and } 5 \text{ ones} = 45 \end{array}$$



After determining which number facts are causing a student to make errors, it will be necessary to provide practice working with the correct number facts. A few minutes practice each day is generally more effective in helping the student than more infrequent and lengthy practice sessions. Practice may be in the form of cross-number puzzles, flash cards, or calculator usage. "Beat the clock" or "beat the calculator" games may be motivational as well as instructional Given exercises which include the troublesome facts, the student may compete with his own time (using a sand timer) or with another student who is using a calculator. In either case, not all problems should focus on multidigit numbers or troublesome facts. Encourage success and confidence as well as practice in the particular number combinations. Combinations which frequently cause difficulties are those whose sums are 14, 15, 16, and 17.

A reversal error may be a lack of understanding of the concept and procedure of subtraction (including the idea of regrouping) or may indicate a very different type problem such as one in visual perception or some other learning disability. If the latter is suspected, it may be necessary to consult with the appropriate counselor or diagnostician as well as the parent and student. If it is a lack of understanding of the concept or procedure, an interview with the student in which he "works out loud" with you may indicate where the trouble is.

Improper alignment of the minuend and subtrahend would probably only occur when a problem is presented in a horizontal format (for example, 839 - 72 = _____.) Practice should include problems written in vertical and horizontal form. However, if students have difficulty with a computational skill, one format should be used consistently until it has been mastered.

## 3.3 MULTIPLICATION OF WHOLE NUMBERS

### Common Errors

1. Making a number fact error.

Examples:

$$\begin{array}{r} 79 \\ \times\ 7 \\ \hline 562 \end{array} \qquad \begin{array}{r} 16 \\ \times\ 29 \\ \hline 135 \\ 32 \\ \hline 455 \end{array}$$

2. Making an error in addition.

Example:

$$\begin{array}{r} 29 \\ \times\ 13 \\ \hline 87 \\ 29 \\ \hline 387 \end{array}$$

3. Improperly Aligning Partial Products

Examples:

$$\begin{array}{r} 66 \\ \times\ 22 \\ \hline 132 \\ 132 \\ \hline 264 \end{array} \qquad \begin{array}{r} 162 \\ \times\ 307 \\ \hline 1134 \\ 486 \\ \hline 5994 \end{array}$$

4. Regrouping incorrectly.

Examples:

$$\begin{array}{r} 206 \\ \times\ 139 \\ \hline 2304 \\ 708 \\ 206 \\ \hline 29984 \end{array} \qquad \begin{array}{r} 38 \\ \times\ 54 \\ \hline 152 \\ 180 \\ \hline 1952 \end{array}$$

5. Using an inappropriate algorithm.

Examples:

$$\begin{array}{r} 42 \\ \times\ 7 \\ \hline 354 \end{array} \qquad \begin{array}{r} 38 \\ \times\ 54 \\ \hline 32 \\ 15 \\ \hline 47 \end{array}$$

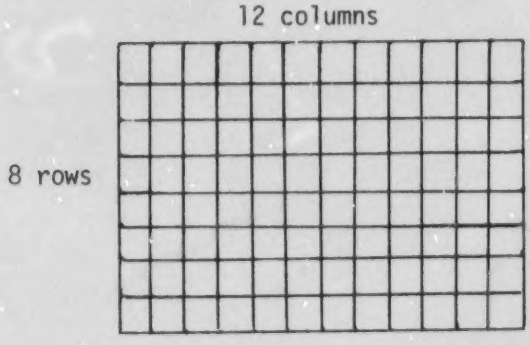
### Instructional Suggestions and Activities

Using an incorrect algorithm may be the result of temporary confusion with another operation or a lack of understanding of the concept of multi-

plication. The former situation may be remedied by reminding the student of the procedure for multiplication.

However, if it appears that the student has little or no understanding of the concept or the procedure, it may be necessary to provide instruction in subskills which lead to the algorithm. This may include providing fundamental experiences in grouping and counting.

Example: Interpet 8 x 12 as 8 sets of 12 objects each and visualize as follows (on a larger scale).




Once the number facts causing difficulty are determined, the student will need to practice the correct combinations. It is generally more effective if practice is provided for short periods of time each day than if it occurs infrequently for longer periods of time. Practice in the form of cross-number puzzles, flash cards, "beat the clock" or "beat the calculator" games can focus on troublesome combinations. In designing exercises for the "beat the clock" or calculator games you should include problems that can be done quickly mentally (such as 10 x 12) so that success and confidence are encouraged as well as practice with particular number combinations. In "beat the clock" a student can compete with himself and try to better his time. (A sand timer works well in class.) In "beat the calculator" games, one student uses paper and pencil or mental arithmetic and another uses the calculator.

Mistakes in addition may be the result of number fact or regrouping errors. For instructional suggestions you may wish to refer to the suggestions listed under "Addition of Whole Numbers".

Both proper alignment of partial products and regrouping depend on an understanding of place value. If students are having difficulty in these areas, emphasis on writing complete partial products, expanded notation and the distributive property, and labeling the place value of columns, may be beneficial.

Specific examples are listed below.

Example:

```
  42
x  7
 14  (7 x 2)
280  (7 x 40)
294
```

or

$$7 \times 42 = 7\,(40 + 2)$$
$$= 7 \times 40 + 7 \times 2$$
$$= 280 + 14$$
$$= 294$$

Example:

```
  142
x  36
  12  (6 x 2)
 240  (6 x 40)
 600  (6 x 100)
  60  (30 x 2)
1200  (30 x 40)
3000  (30 x 100)
5112
```

or

```
  142
   36
 852  (6 x 142)
4260  (30 x 142)
5112
```

Zeros in either factor, especially intermediate zeros, frequently cause problems. Again stressing place value and writing complete partial products may be helpful.

Example:

```
  304
x   3
  12  (3 x 4)
   0  (3 x 0)
 900  (3 x 300)
 912
```

```
  167
  203
 501  (3 x 167)
 000  (0 x 167)
33400  (200 x 167)
33901
```

As students become more proficient and confident, encourage them (if they don't do it on their own) to use the shorter form. You may wish to vary the type of practice you give the students by asking them to supply missing digits which will make a multiplication problem complete and correct.

Examples:

```
  27
x  -
 --8
```

```
  25
x --
 100
 -5
---
```

## 3.4 DIVISION OF WHOLE NUMBERS

### Common Errors

1. Multiplying incorrectly.
2. Subtracting incorrectly.
3. Omitting digits in the quotient.

Example:

```
   28
6)1248
  12
  ---
   48
   48
  ---
```

4. Improperly placing digits in the quotient.

Example:

```
   280
6)1248
  12
  ---
   48
   48
  ---
```

5. Incomplete division.

Example:

```
   79 R 53
6)527
  42
  ---
  107
   54
  ---
   53
```

### Instructional Suggestions and Activities

If students are making errors in multiplication or subtraction, you may wish to refer to suggestions under skills 3.2 and 3.3.

Before a student can be successful in performing division computations, he must have an understanding of the meaning of division and of place value, be able to estimate in cases such as 5 x (10, 100, or 1000) ≐ 586 and be able to supply missing factors such as 6 x _____ = 54.

If students are making errors by omitting digits in the quotient or in placing them in the quotient, stress writing neatly and place value alignment of digits in the quotient with those in the dividend. It may be beneficial to have them work on vertically lined paper or grid paper. Practice in estimating quotients, naming the place value of the first digit of the

quotient before dividing, and verifying their results may also be helpful.

If a student's errors are the result of not understanding the concept of division, consideration should be given to using a method of division similar to the one described below.

Example: Consider $586 \div 5$.

(The questions and answers suggested are intended to be interactions between student and teacher although they are not written as script dialogue here. The "stacking" of numbers in the quotient described in the text is illustrated at the beginning so that you may refer to it as you read.)

$$
\begin{array}{r}
117 \text{ R } 1 \\
7 \\
10 \\
100 \\
5\overline{)586} \\
500 \\
\hline
86 \\
50 \\
\hline
36 \\
35 \\
\hline
1
\end{array}
$$

How many sets of 5 objects can we make from 586 objects? As many as 100? Yes, because 5 x 100 = 500 and this is less than 586. Therefore, write 100 in the quotient. Do we have any left? Yes, how many? 86. How many sets of 5 objects can we make from those 86 objects? As many as 20? No. Why not? Can we make as many as 10? Why? Because 5 x 10 = 50 and 50 is less than 86. Write 10 above the hundred and align the place values in the quotient. Do we have any left? How many? 36. How many sets of 5 objects can we make from those 36? 7 because 7 x 5 = 35. Write 7 above the 10 and align it in the ones' place. Now do we have any left? Yes. 1. Can we make any more sets of 5 objects? No, because we only have 1 left. Then how many sets of 5 objects have we made? 100 + 10 +7 = 117 sets and we have 1 that is not in any set.

## 4.1 ADDITION AND SUBTRACTION OF DECIMAL FRACTIONS

### Common Errors

1. Making an addition or subtraction error as described under "Addition of Whole Numbers" and "Subtraction of Whole Numbers".

   Example:
   $$\begin{array}{r} 15.43 \\ -\ 2.06 \\ \hline 13.47 \end{array}$$

2. Improperly aligning the decimal points or incorrectly placing the decimal point in the sum or difference.

   Example: $1.72 + .6 =$ _____
   $$\begin{array}{r} 1.72 \\ +\ .6 \\ \hline 1.78 \end{array}$$

   $1.25 + .7 =$ _____
   $$\begin{array}{r} 1.25 \\ +\ .700 \\ \hline .825 \end{array}$$

   $32.7 - 2.35 =$ _____
   $$\begin{array}{r} 32.7 \\ -\ 2.35 \\ \hline 0.92 \end{array}$$

   $0.8 + 0.7 =$ _____
   $$\begin{array}{r} 0.8 \\ +\ 0.7 \\ \hline 0.15 \end{array}$$

3. Adding the terminating digit(s) of the subtrahend when the subtrahend has more decimal places than the minuend.

   Example:
   $$\begin{array}{r} 0.28 \\ -\ 0.104 \\ \hline 0.184 \end{array}$$

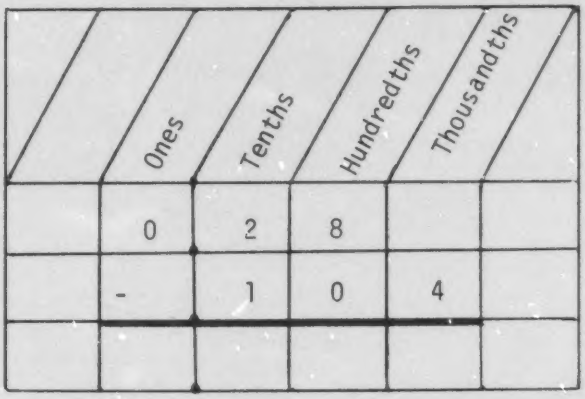
### Instructional Suggestions and Activities

If students are making errors which relate specifically to whole number computation, you may wish to refer to the suggestions listed in "Addition of Whole Numbers" or "Subtraction of Whole Numbers".

The addition error described above in error 3, as well as improper alignment of decimal points, may be the result of a lack of understanding of place value, equivalent decimals, or the necessity of aligning digits according to place value position. Visual representations on area grids may promote understanding of the concept of equivalent decimal fractions

(for example, .5 = .50 = .500). It may be beneficial to encourage students to annex zeros so that each numeral in a problem has the same number of digits. Stress that careful alignment of decimal points assists in aligning place value positions and vice versa. Allowing students to work on grid paper or use charts similar to the one below may help them focus on proper alignment of both.

Example: 0.28 - 0.104 = ____




## 4.2 MULTIPLICATION OF DECIMAL NUMBERS

### Common Errors

1. Improperly placing the decimal point in the product.

Examples:

| 16.5 | .04 | .179 |
|---|---|---|
| x .5 | x .3 | x .11 |
| 82.5 | .12 | .19690 |

Note: Although other errors may occur in multiplying decimal numbers, to distinguish between this skill and multiplication of whole numbers, the focus is on placement of the decimal point in the product.

## Instructional Suggestions and Activities

If students are making multiplication errors which reflect deficiences in whole number multiplication, you may wish to refer to the suggestions listed under "Multiplication of Whole Numbers". Stress that the number of digits to the right of the decimal point in the product is the sum of the number of digits to the right of the decimal point in the two factors. Practice may be in the form of completing a chart similar to the one shown below or placing the decimal point in the product given the factors and the digits of the product.

Examples: Complete the Chart

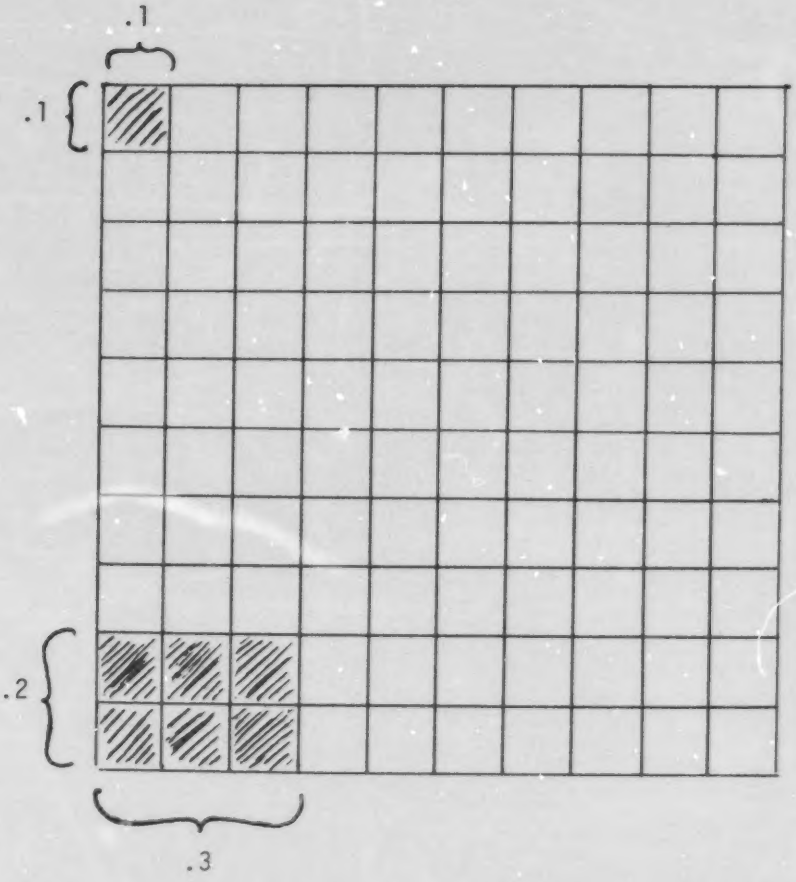
| | Number of Decimal Places | | |
|---|---|---|---|
| Factors | First Factor | Second Factor | Product |
| 8 x .6 | 0 | 1 | 1 |
| .9 x .6 | 1 | 1 | - |
| .05 x .3 | 2 | 1 | - |
| .12 x .072 | - | 3 | 5 |
| .82 x 10 | 2 | - | - |

Place the Decimal Point

12.3 x .35 =
a) .4305
b) 4.305
c) 43.05
d) 430.5

Additional instruction may be necessary in situations where students must insert zeros as place holders to obtain the correct number of digits in the product (for example, .2 x .3 = .06). If difficulty in this type of problem appears to be the result of a lack of understanding of the concept of a decimal fraction, it may be beneficial to provide visual examples and experiences using area grids.

Example: The shaded square at the top left of the grid represents one hundredth of the area of the grid. The shaded rectangle at the bottom left of the grid represents six hundredths of the area of the grid.




## 4.3 DIVISION OF DECIMAL NUMBERS

### Common Errors

1. Improperly placing the decimal point in the quotient.

   Examples:

$$\begin{array}{r} .18 \\ 7\overline{).126} \end{array} \qquad \begin{array}{r} 3. \\ .75\overline{)22.5} \end{array}$$

$$\begin{array}{r} 2.1 \\ 4\overline{)8.04} \end{array} \qquad \begin{array}{r} .5 \\ .7\overline{)3.5} \end{array}$$

   Note: Although other errors may occur in dividing decimal numbers, to distinguish between this skill and division of whole numbers, the focus here is on placement of the decimal point in the quotient.

## Instructional Suggestions and Activities

If students are making errors which reflect deficiencies in whole number division, you may wish to refer to the suggestions listed under skill 3.4.

Improper placement of the decimal point in the quotient may be the result of a misconception about the procedure of changing the problem to an equivalent one having a whole number divisor. Consideration should be given to helping students understand the concept which underlies the practice of "moving" the decimal point in the dividend and the divisor. That is, their "moving" the decimal point in the dividend the same number of places to the right as they "move" it in the divisor is equivalent to multiplying the divisor and the dividend by the same multiple of 10. It is important to stress that the divisor must be a whole number.

Using money examples or metric units of measurement may be helpful in illustrating division of decimals. However, ample opportunity should be provided for students to practice correctly placing the decimal point in the quotient when given the dividend, the divisor, and the digits of the quotient.

Examples: Divide: $7\overline{).021}$

a) 30
b) .03
c) .003
d) 3.0

Place the decimal point in the answer to this problem to make it correct.

$$2\overline{)3.12}\quad 12$$

## 5.1 FINDING THE PRODUCT OF TWO SIMPLE FRACTIONS

### Common Errors

1. Makes computational errors.

   Example: $\frac{1}{4} \times \frac{1}{3} = \frac{1}{15}$

2. Multiplies the numerator of the first fraction by the denominator of the second and multiplies the denominator of the first fraction by the numerator of the second.

Example: $\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 4}{2 \times 3} = \frac{4}{6}$

or

$\frac{1}{2} \times \frac{3}{4} = \frac{2 \times 3}{1 \times 4} = \frac{6}{4}$

3. Finds a common denominator and then multiplies only the numerators.

   Example: $\frac{1}{2} \times \frac{1}{3} = \frac{3}{6} \times \frac{2}{6} = \frac{6}{6}$ or 1

4. Adds the numerators and/or denominators.

   Example: $\frac{1}{2} \times \frac{1}{3} = \frac{2}{6}$

5. Multiplies the first fraction by the reciprocal of the second one or vice versa.

   Example: $\frac{1}{2} \times \frac{1}{3} = \frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$

   or

   $\frac{1}{2} \times \frac{1}{3} = \frac{2}{1} \times \frac{1}{3} = \frac{2}{3}$

6. Simplifies the product incorrectly.

   Example: $\frac{1}{\cancel{2}\ 2} \times \frac{\cancel{4}}{5} = \frac{1}{10}$

   $\frac{2}{3} \times \frac{1}{4} = \frac{\cancel{2}}{\cancel{12}\ 6} = 6$

Instructional Suggestions and Activities

If a student makes computational errors (Error 1) a review of whole number algorithms is needed. See the appropriate skill descriptions for suggested instructional activities.

If students find common denominators and then multiply the numerators (Error 3) or find the reciprocal of one fraction before multiplying both numerators and denominators, they may only be confusing the multiplication algorithm with those of addition and division, respectively. Thus, reviewing all the algorithms together may be what is needed to correct the errors. However, the errors may

indicate a lack of concept development as far as multiplication of fractions is concerned. Hence, these errors as well as Errors 2, 4, and 6 may result from a lack of understanding of concepts that form a basis for the development of the algorithm.

An instructional procedure that may assist in teaching students to multiply fractions is given on the next several pages. Its goals are to:

(1) Have students understand what multiplication of fractions means by developing computational algorithms through the use of concrete materials.

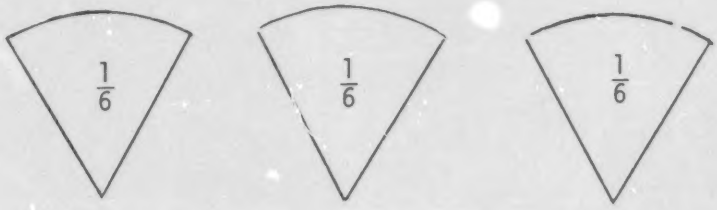
(2) Use what the students already know about multiplication of whole numbers and extend those items to include fractional quantities.

## Multiplication of Fractions

Before dealing with multiplication of fractions, we need to agree on what simple fractions mean. What do the numerals mean? The following interpretation is usually made:

$$\frac{1}{4} \leftarrow \frac{\text{1 part of}}{\text{4 equal parts}}$$

Look at the 3 pieces below. What fraction could we assign to those three pieces combined? (Answer: 3/6)




Consider the people in this room. How could we, without multiplying, put 2/3 of the group of people in one place? We could sort the people, 1 by 1, into three equal groups and then put two of those three equal groups (2/3) together. We will use this same idea throughout our work with the circular sections.

Let's use these fraction ideas to work with multiplication of fractions. We have decided to build on the multiplication of whole numbers.

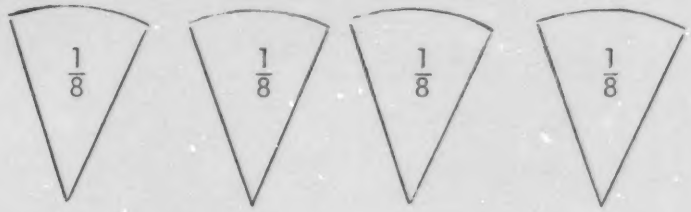
In the problem 3 x 4 = _____, what do the numerals 3 and 4 mean? Generally, 3 x 4 is interpreted to mean 3 sets of 4; that is, 3 indicates the number of sets and 4 indicates the size of each set.

For the multiplication of fractions, the fractional numbers will have

identical meanings to their corresponding whole numbers problems.

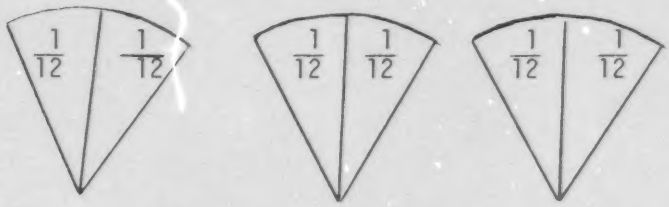
Specifically, if $\triangle \times \triangledown = o$
$\triangle$ indicates the number of sets, and
$\triangledown$ indicates the size of each set.

Let's try 4 x 1/8 = _____.



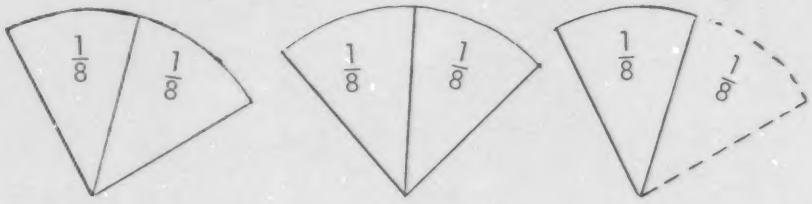

4 sets of 1/8 = 4/8 or 1/2

Now let's do 3 x 2/12 = _____.




So 3 sets of 2/12 = 6/12 or 1/2.

Now let's try 2½ x 2/8 = _____.



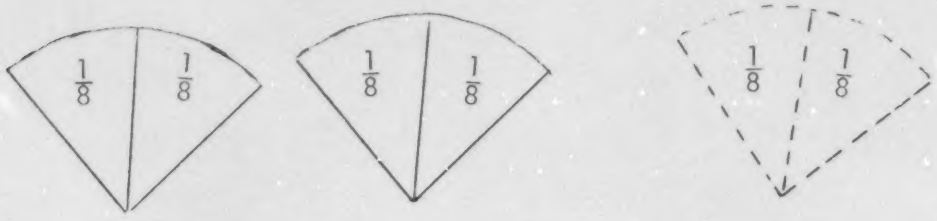

1 set of 2/8 1 set of 2/8 1/2 set of 2/8

So 2½ sets of 2/8 = 5/8.

NOTE: To find ½ set of 2/8 we put the 2/8 into two equal groups (1/8 in each group) and use 1 of those two equal groups.

Here's a different one, 2/3 x 6/8 = _____.

First we take out a set of 6/8. Our problem tells us that we want 2/3 sets of 6/8. How do we find 2/3 sets? Remember how we found 2/3 of the people in this room? That's right, we put our objects (6/8) into 3 equal groups and use two of those three equal groups (2/3).




2 of 3 equal groups

Thus, 2/3 x 6/8 = 4/8 or 1/2.

Now, let's focus on some problems that will generalize nicely to give us a computational algorithm for multiplying fractions.

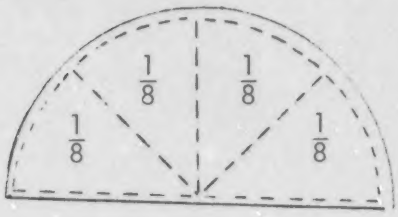
1) 1/2 x 1/3 = _____.

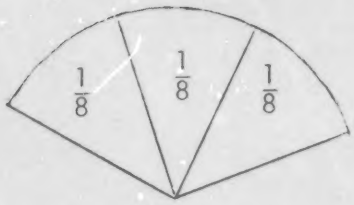
1 of 2 equal parts

Dividing 1/3 into two equal parts is easily done by finding two identical circular sections which together exactly cover 1/3. One of those two equal pieces then represents 1/2 of the 1/3 piece.

Thus, 1/2 x 1/3 = 1/6.

2) 3/4 x 1/2 = _____.




3 of 4 equal parts

Thus, 3/4 x 1/2 = 3/8.

On the basis of these three examples it looks like:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d} \text{ or } \frac{ac}{bd}$$

## 6.1 RENAMING PERCENTS AS DECIMAL FRACTIONS

### Common Errors

1. Improperly placing the decimal point in the decimal fraction.

   Examples: 2% written as 200.

   2% written as .20

   2% written as .0002

2. Making an error in the process of renaming the percent as a fraction and the fraction as a decimal.

### Instructional Suggestions and Activities

Improper placement of the decimal point in the decimal fraction may be the result of confusion about the meaning of percent or a lack of understanding of place value. It is important to stress the meaning of percent as a special ratio, i.e., a given percent indicates the specified number per hundred. It may be helpful to use an area grid of 100 square units to show visual representations of various percents (including 100%). Discussing the use of percent to describe situations, such as 30 out of 100 people voted for a particular candidate, may also be beneficial. It is important

that students feel confident about the meaning of percent before being required to rename percents as decimal fractions. Instruction in renaming percents should then focus on place value and percents which are specified by whole numbers between 0 and 100. Percents specified by mixed numerals, percents less than one and percents greater than 100 are not considered a part of the minimum competencies.

An error in the process of renaming a percent as a common fraction and the fraction as a decimal may be the result of confusion about the meaning of percent, an inappropriate procedure or a computational error. The suggestions stated in the previous paragraph relative to emphasis on the meaning of percent are also appropriate here. Suggestions relative to renaming a common fraction as a decimal are listed under skill 7.1.

## 7.1 EXPRESSING COMMON FRACTIONS AS DECIMAL FRACTIONS

### Common Errors

1. Making an error in computation.

   Example: Writing $\frac{5}{8}$ as .725 and showing the division as

$$
\begin{array}{r}
.725 \\
8\overline{)5.000} \\
48 \\
\hline
20 \\
16 \\
\hline
40 \\
40 \\
\hline
\end{array}
$$

2. Dividing the denominator by the numerator.

3. Incorrectly placing the decimal point in the decimal fraction.

   Example: Writing $\frac{5}{8}$ as 62.5.

### Instructional Suggestions and Activities

If a student is incorrectly placing the decimal point in the decimal fraction or making an error in computation you may wish to refer to the suggestions listed under skills 3.4 and 4.3.

Dividing the denominator by the numerator may be the result of a lack of understanding of the procedure required to rename common fractions as decimals or of the concept of fractions. If it is the former, stressing that the bar or line segment in the fraction symbol indicates "divided by"

and that the denominator is the divisor may be helpful. In both cases, consideration may be given to using visual representations to illustrate equivalent fractions.

Example: $\frac{3}{5}$

.6

Practice may be in the form of cross-number puzzles, "beat the clock" or "beat the calculator" games. In such games the student either competes with his own time or with another student who is using a calculator. In either case, equivalents which students may remember as well as those that may require computation, should be included.

Not restricting the focus to troublesome equivalents encourages success and helps the student gain confidence. Activities to discover patterns in decimal equivalents for fractions with the same denominators may create interest as well as practice in working with repeating decimals.

For example: $\frac{1}{9} = .\overline{1}$, $\frac{2}{9} = .\overline{2}$, $\frac{3}{9} = .\overline{3}$, $\frac{4}{9} = .\overline{4}$, $\frac{5}{9} = .\overline{5}$, $\frac{6}{9} = .\overline{6}$,

$\frac{7}{9} = .\overline{7}$, $\frac{8}{9} = .\overline{8}$

## 8.1 COMPUTING PERCENTAGE

### Common Errors

1. Dividing by the decimal equivalent of the percent to compute the percentage.

2. Incorrectly expressing the decimal or common fraction equivalent.

   Example: Expressing 2% as .2 or $\frac{2}{10}$

3. Multiplying incorrectly.

### Instructional Suggestions and Activities

Before a student is able to compute percentages he must know that the required operation is multiplication as well as know how to write the percent as an

equivalent fraction. An example using money may help illustrate that the required operation is multiplication. Consider finding 20% of $4.00 as 20 per hundred of $4.00 or 20 per hundred of 4 hundred pennies. And this is 80 (20 x 4) pennies. Students should be allowed to work with the decimal or common fraction equivalent of the percent, depending on which method they are more proficient in using. If they appear to have difficulty in both methods, you may wish to refer to suggestions listed under skill 6.1.

If a student multiplies incorrectly, it may be the result of an error in multiplication of whole numbers, decimals, or common fractions. Instructional suggestions are listed under skills 3.3, 4.2, and 5.1.

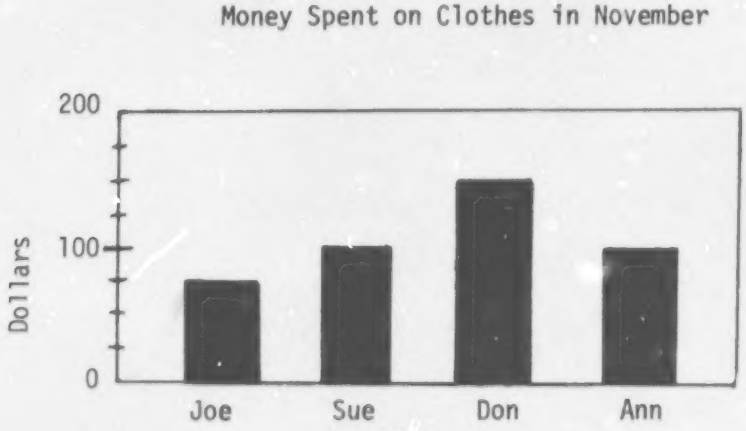
Newspaper articles, survey results and the like, provide interesting sources of problems for practice. For example, election results reported as percents will require the student to compute percentages to determine the number of people voting for each candidate. (They must also be given the number of people voting!)

## 9.1 READING BAR AND BROKEN-LINE GRAPHS

### Common Errors

1. Reading the number scale(s) incorrectly.

   Example: Reading, from the graph below, the amount Joe spent on clothes as $90 rather than $75.




2. Incorrectly aligning the end of a bar on a bar graph or a point on a broken-line graph with its corresponding scale(s) value.

   Example: From the graph in (1), a student may read the amount Ann spent on clothes as $75.

3. Making a computational error when asked to compare quantitative information.

   Example: Using information on the graph in (1), a student may state, as the result of a subtraction error, that Don spent $85 more than Joe.

## Instructional Suggestions and Activities

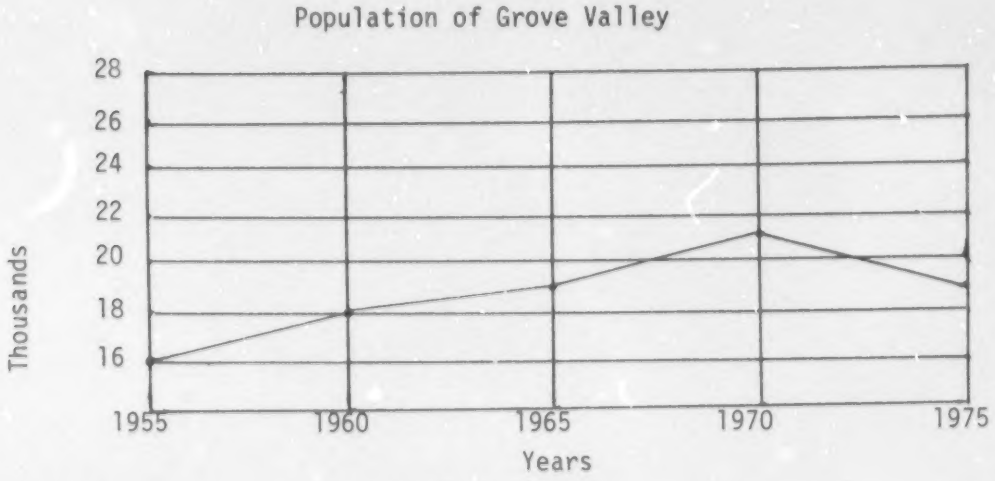
Reading scales on graphs frequently causes confusion for students. Ample opportunity to practice reading various numerical scales should be provided. It may be necessary to practice reading various scales along a horizontal line before reading them on a graph. Determining the scale as well as estimating intermediate values should be emphasized. Encouraging students to use a straight edge to align the end of a bar on a bar graph or a point on a broken-line graph with its appropriate scale value may be helpful. In addition to reading scales, consideration should be given to careful reading of the title of each graph as well as the information represented by each axis.

It may be necessary to provide instruction and practice in locating given points on a rectangular grid and in naming locations of given points on a grid. Interest may be greater if the grids represent seating charts, street maps (horizontal and vertical streets), or the like. Consideration may also be given to asking questions which may require computation and comparison.

Example: Using information on the graph below, determine the following:

a) How many more people lived in Grove Valley in 1965 than in 1955?

b) The population of Grove Valley was highest during which year?



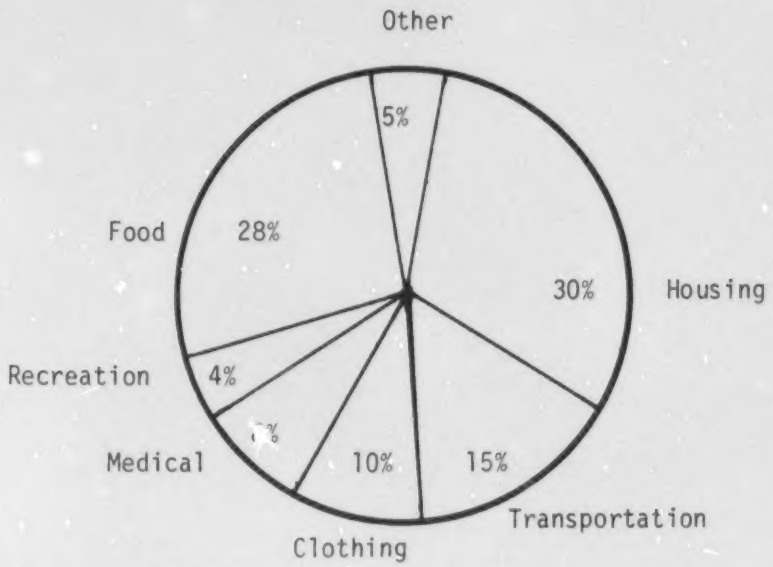
Instruction is enhanced if the data used is of interest to the students. Involving them in collecting data, organizing it, and/or displaying it may be motivational. Topics to consider are time spent per week watching television, pulse rate after running, birth months of students and their family members, school sports data, or time spent traveling to school. Bringing in graphs from current magazines and newspapers may also provide interest.

## 9.2 READING CIRCLE GRAPHS

### Common Errors

1. Adding incorrectly when asked to compare the sum of two sectors with one or more sectors.

   Example:




   Referring to the circle graph the student may state, as the result of an addition error, that food and transportation account for more than 50% of the total budget.

2. Confusing terms of comparison such as "twice as much" and "one-half as much".

   Example: Referring to the graph shown in error 1, students may respond that food accounts for half as much of the budget as recreation and food combined.

Instructional Suggestions and Activities

In studying circle graphs, students must understand that the circle represents a whole entity and each sector represents a part of that whole. Generally, the title and several support statements will describe what the circle is representing. Therefore, it is important for students to read this information carefully. Likewise, it is important for them to name the indicated percent and title of each mutually exclusive sector of the circle.

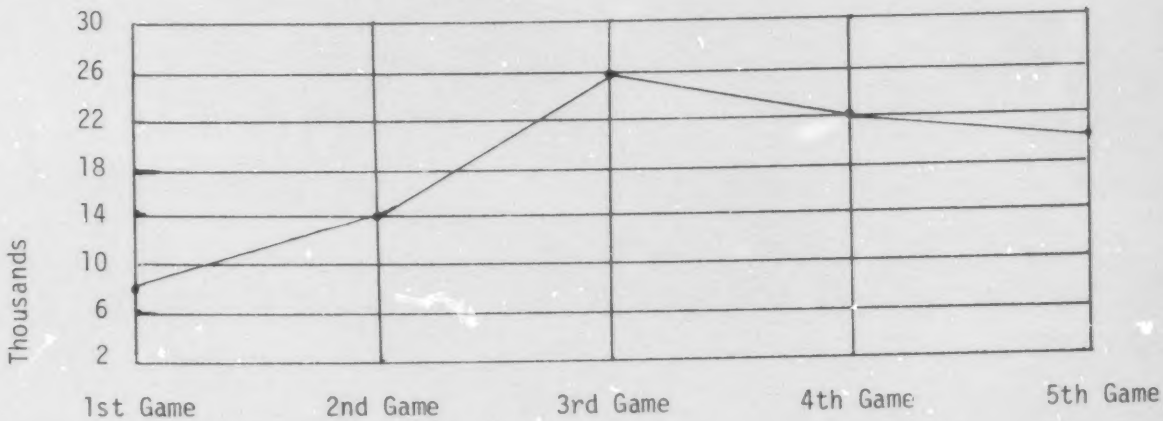
Once students have demonstrated that they are able to read the information on a circle graph, instruction may focus on understanding the words of comparison as well as using computation in comparing quantitative information. Students must be able to distinguish between comparative phrases such as "twice as much" and "one-half as much". Computation may involve comparing the sum of two sectors with one or more sectors. Errors in computation may be the result of deficiencies in computational skills. Suggestions for instruction in whole number computation are listed under skills 3.1 through 3.4.

## 9.3 INTERPRETING GRAPHS

Common Errors

1. Confusing words of comparison, such as most and least and increase and decrease.
2. Reading a number scale(s) incorrectly. For examples, refer to the errors listed under skill 9.1.
3. Making a computational error.

Example:



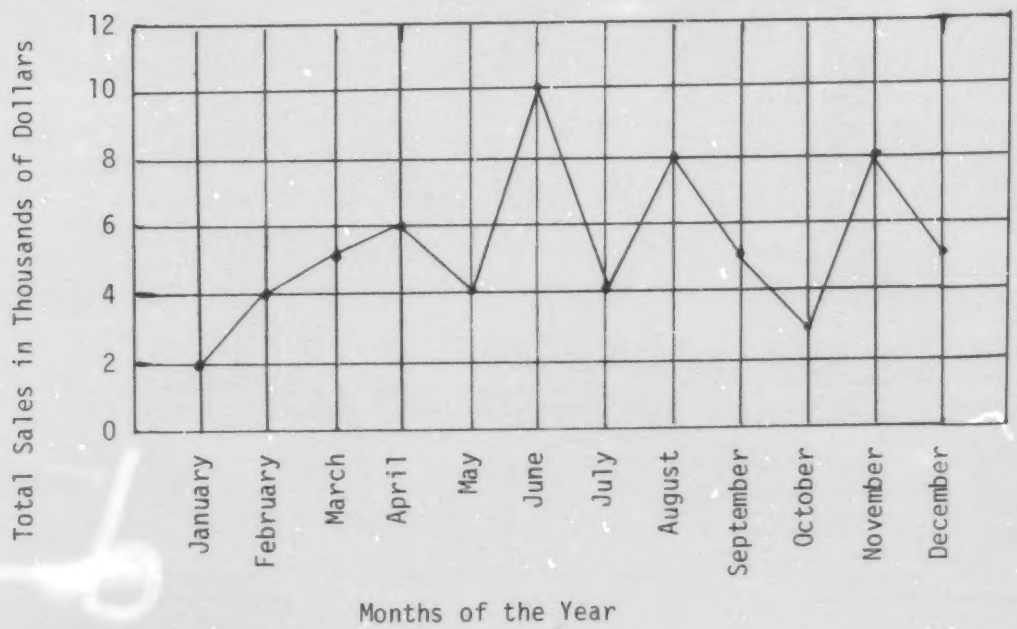
Referring to the broken-line graph, a student may say, because of a subtraction error, attendance decreased by 9,000 from the third game to the fourth game.

## Instructional Suggestions and Activities

Many illustrations and discussions of graphs of a variety of data are beneficial to students. In each case, reading the title, identifying the scale(s) where appropriate, and reading basic information from the graph should precede making comparisons of size or frequency and discussing trends of increase, decrease, or constancy. Suggestions relative to instruction in reading graphs are listed under skills 9.1 and 9.2.

In discussing information on graphs, frequent use of such words as most, least, decrease, increase, decline, more, or less, is important. Encourage students to look for extreme values and trends of increase, decrease, or constancy where appropriate. Discuss possible reasons for these trends or abrupt changes. It may be necessary to focus on the concept of increase or decrease through examples, such as the one shown below. In particular, students may have difficulty identifying trends of increase (decrease) when the rate of increase (decrease) is not constant. An example is the sales increase from January to March in the following graph.

Example: Manufacturing Company's Monthly Sales for 1976



According to the graph, which statement is true?

a) Company sales decreased from October to December
b) Company sales increased from May to June
c) Company sales remained constant from January to March
d) Company sales were the same amount for April and November

Instruction is enhanced if the data used is of interest to the students. Involving them in collecting, organizing or displaying it may also be motivational. Topics may include birth months of students and their families, time spent per week watching television, personal budgets, temperature in the classroom each day, time spent traveling to school, or school sports. Bringing in graphs from current magazines or newspapers or collecting graphs of how various charities, businesses or governmental agencies spend their money may also provide interest.

## 10.1 READING A MAP

### Common Errors

1. Not using all of the pertinent information shown on the map, i.e., the student focuses on the route numbers and ignores indicated distances between locations.

2. Making an error in adding the appropriate mileages.

3. Using inappropriate addends, for example, adding route numbers or route numbers and mileages.

### Instructional Suggestions and Activities

Instruction should focus on locating points on a map, identifying alternate routes between two locations, reading mileages indicated along routes, and determining routes of least mileage. Using maps of cities of interest to the students may be motivational. This may include places they would like to visit or have visited as well as cities, parks, or museums within a given radius of their hometown.

Ample opportunity for practice in map reading must be provided. A student is more likely to carefully look at all information on a map if he is familiar with the type of information generally displayed. Careful attention to reading may also help a student avoid making errors resulting from adding route numbers and mileages or adding route numbers rather than mileages.

## 11.1 FINDING THE AMOUNT OF INCOME TAX FROM A TAX TABLE

### Common Errors

1. Uses the incorrect row or column in a table to determine the amount of tax.

   Examples: a) A student may not follow the correct row or column.

   b) A student may not know which column to use because of a lack of understanding of terminology such as marital status or number of dependents.

2. Compares numerical values incorrectly.

   Examples: a) A student may not know that an adjusted gross income of $10,010 falls within the interval from $10,001 to $10,100.

   b) A student may not understand such terminology as "at least... but less than ...".

### Instructional Suggestions and Activities

Show students how to use straight edges along the row and column to locate an entry on the table:

Partial Tax Table

| If line 47 (taxable income) is— | | And you are— | | | | If line 47 (taxable income) is— | | And you are— | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Over | But not over | Single | Married filing separately | Head of a household | Married filing jointly * | Over | But not over | Single | Married filing separately | Head of a household | Married filing jointly * |
| | | Your tax is— | | | | | | Your tax is— | | | |
| 8,550 | 8,600 | 1,734 | 1,791 | 1,[illegible] | 1,507 | 11,750 | 11,800 | 2,569 | 2,758 | 2,384 | 2,211 |
| 8,600 | 8,650 | 1,746 | 1,805 | 1,[illegible] | 1,518 | 11,800 | 11,850 | 2,583 | 2,774 | 2,396 | 2,222 |
| 8,650 | 8,700 | 1,759 | 1,819 | 1,[illegible] | 1,529 | 11,850 | 11,900 | 2,596 | 2,790 | 2,409 | 2,233 |
| 8,700 | 8,750 | 1,771 | 1,833 | 1,[illegible] | 1,540 | 11,900 | 11,950 | 2,610 | 2,806 | 2,421 | 2,244 |
| 8,750 | 8,800 | 1,784 | 1,847 | 1,658 | 1,551 | 11,950 | 12,000 | 2,623 | 2,822 | 2,434 | 2,265 |
| 8,800 | 8,850 | 1,796 | 1,861 | 1,670 | 1,562 | 12,000 | 12,050 | 2,637 | 2,839 | 2,447 | 2,266 |
| 8,850 | 8,900 | 1,809 | 1,875 | 1,681 | 1,573 | 12,050 | 12,100 | 2,652 | 2,857 | 2,460 | 2,279 |
| 8,900 | 8,950 | 1,821 | 1,889 | 1,693 | 1,584 | 12,100 | 12,150 | 2,666 | 2,875 | 2,474 | 2,291 |

Then provide practice opportunities reading various tax tables -- using the straight edges as guides.

Explain the meaning of various terms used as column headings in tax tables. This would include such terms as: exemptions; dependents; adjusted gross



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income; single, not head of a household; married, filing joint return; etc. You may want to put the students in various role-playing situations as you are discussing the meaning of the terms. This provides practice in using the terms and is much more practical than having students memorize "dictionary" definitions of the terms.

You may choose an adjusted gross income interval on the tax table, such as "$10,001 to $10,100" and have the student list the various amounts of income that fall within the interval.

Review the meaning of such terms as "over ... but not over","at least ... but less than". Terms such as these are used to locate an amount of tax on a given adjusted gross income. You may want to call out some adjusted gross incomes and have students call out the amount of tax.

## 11.2 FINDING THE AMOUNT OF SALES TAX FROM A TAX TABLE

### Common Errors

1. Computes the total cost of several items incorrectly before determining sales tax from the table.

2. Uses the incorrect row or column in a table to determine the amount of sales tax on a given purchase.

3. Compares numerical values incorrectly.

   Example: A student may be asked to find the sales tax on a $3.20 purchase. This amount of purchase is not on the table. So a student must decide which two entries this amount is between in order to find the sales tax. (S)he may not know that $3.20 falls within the interval from $3.15 to $3.34.

### Instructional Suggestions and Activities

See sections on addition computation for suggested instructional activities.

Show students how to use straight edges along a row and column to locate an entry on a table.

PARTIAL TAX TABLE

**4% VIRGINIA SALES TAX TABLE**

FOR COMBINED STATE TAX AT THE RATE OF 3% AND LOCAL TAX AT THE RATE OF 1%

**FOR USE ON AND AFTER JULY 1, 1968**

| Amount of Sale | Tax | Amount of Sale | Tax | Amount of Sale | Tax |
|---|---|---|---|---|---|
| .01 — .14 | .00 | 14.88 — 15.12 | .60 | 29.88 — 30.12 | 1.20 |
| .15 — .34 | .01 | 15.13 — 15.37 | .61 | 30.13 — 30.37 | 1.21 |
| .35 — .59 | .02 | 15.38 — 15.62 | .62 | 30.38 — 30.62 | 1.22 |
| .60 — .84 | .03 | 15.63 — 15.87 | .63 | 30.63 — 30.87 | 1.23 |
| .85 — 1.14 | .04 | 15.88 — 16.12 | .64 | 30.88 — 31.12 | 1.24 |
| 1.15 — 1.34 | .05 | 16.13 — 16.37 | .65 | 31.13 — 31.37 | 1.25 |
| 1.35 — 1.59 | .06 | 16.38 — 16.62 | .66 | 31.38 — 31.62 | 1.26 |
| 1.60 — 1.84 | .07 | 16.63 — 16.87 | .67 | 31.63 — 31.87 | 1.27 |
| 1.85 — 2.14 | .08 | 16.88 — 17.12 | .68 | 31.88 — 32.12 | 1.28 |
| 2.15 — 2.34 | .09 | 17.13 — 17.37 | .69 | 32.13 — 32.37 | 1.29 |
| 2.35 — 2.59 | .10 | 17.38 — 17.62 | .70 | 32.38 — 32.62 | 1.30 |
| 2.60 — 2.84 | .11 | 17.63 — 17.87 | .71 | 32.63 — 32.87 | 1.31 |
| 2.85 — 3.14 | .12 | 17.88 — 18.12 | .72 | 32.88 — 33.12 | 1.32 |
| 3.15 — 3.34 | .13 | 18.13 — 18.37 | .73 | 33.13 — 33.37 | 1.33 |
| 3.35 — 3.59 | .14 | 18.38 — 18.62 | .74 | 33.38 — 33.62 | 1.34 |
| 3.60 — 3.84 | .15 | 18.63 — 18.87 | .75 | 33.63 — 33.87 | 1.35 |
| 3.85 — 4.14 | .16 | 18.88 — 19.12 | .76 | 33.88 — 34.12 | 1.36 |
| 4.15 — 4.34 | .17 | 19.13 — 19.37 | .77 | 34.13 — 34.37 | 1.37 |
| 4.35 — 4.59 | .18 | 19.38 — 19.62 | .78 | 34.38 — 34.62 | 1.38 |
| 4.60 — 4.84 | .19 | 19.63 — 19.87 | .79 | 34.63 — 34.87 | 1.39 |

Have students make some purchases from catalogs or newspaper ads. Then have them use a sales tax table to find the correct amount of tax. This type of activity not only provides practice for reading tables and charts, but also serves as a motivational situation to students.

Review with students how to interpret the "amount of purchase" column (EX: $20.26 - $20.49) on a sales tax table. You may want to choose a particular table entry and have the students list each amount of purchase within the interval on which the indicated tax is charged.

## 12.1 IDENTIFYING PARALLEL LINES

### Common Errors

1. Confuses the word parallel with the word perpendicular.

2. Confuses the concept of parallel lines with that of non-parallel lines.

### Instructional Suggestions and Activities

If students are confusing the words parallel and perpendicular, it may be helpful to point out that the symbol for parallel ("||") looks very much

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like the two l's written next to each other in the word parallel. This association may keep students from confusing the two words.

Use paperfolding activities to aid in identification of parallel lines. These activities can be used to distinguish parallel lines from non-parallel lines.

Ask students to look around the classroom to find as many examples of parallel lines as they possibly can.

Provide students with a map of both rural and city areas. Have them examine the use of parallel and non-parallel lines. (NOTE: They should find a greater number of intersecting lines in city street plans.)

## 12.2 IDENTIFYING THE CENTER, A RADIUS, AND A DIAMETER OF A CIRCLE

### Common Errors

1. Identifies a radius as a diameter or vice versa.
2. Confuses the center, a radius, and a diameter with other points or line segments within or on the circle.
3. Confuses the circumference with the diameter or other parts of a circle.

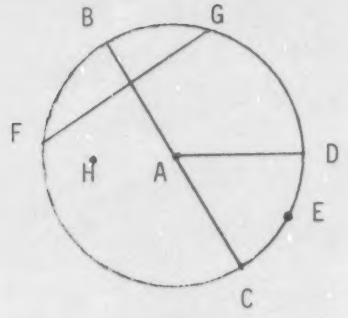
### Instructional Suggestions and Activities

The terms radius and diameter are used in many real-world situations. Have students relate the use of the words in real-world situations to the meaning of the words as parts of a circle.

Example: "The noise can be heard within a fifty mile radius of downtown Richmond".

Use paperfolding activities to aid in the identification of parts of a circle. Also, so that students will be sure to recognize the parts of a circle when presented with many points and segments within or on a circle, be sure to have students solve the following type problem:

Example:




Use the diagram to indicate whether line segment BC is:

a) center
b) radius
c) circumference
d) diameter

You may want to point out that many of the letters in the word circle are contained in the word circumference. This may help the student to remember that the circumference is the distance around the circle.

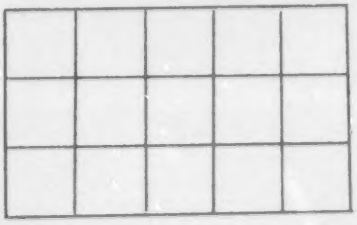
## 13.1 FINDING THE AREA OF A RECTANGULAR REGION

### Common Errors

1. Makes computational errors.

2. Counts points of intersection rather than units of length or square units when the rectangular region is displayed on a grid.

   Example: A student may incorrectly indicate that the area of the following figure is 24 square units.



3. Uses an incorrect formula.

   Example: A student may add the length and width to find the area when, in fact, (s)he needs to multiply the length times the width.

4. Finds the perimeter of a rectangular region rather than its area.

Instructional Suggestions and Activities

Review computational algorithms as needed. See appropriate instructional suggestions included with the computational skill descriptions.

It is important that the area concept be developed through the use of visual aids. Substantial hands-on experiences with physical models of geometric shapes may be needed. Many pupils are not intuitively familiar with the concept of area. They may need experience measuring, partitioning, and counting square units, prior to using formulas.

In order to emphasize that the area of a region is the number of square units that cover a region, have students use square units, such as □ , to cover the space inside a given rectangular region and count the number of square units within the region. This total number of square units within the figure is the area of the rectangular region. (NOTE: A geoboard can be used to make the rectangles.)

Example:

Square Unit

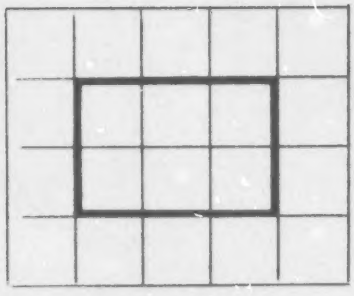
Rectangle

Rectangle with square units covering it

If you count the number of square units covering the rectangular region, you get an area of 8 square units.

A transition must be made from counting square units to find area to determine the length and width of a region and then multiplying to find the area.

Example: Give students a specific area measurement and have them draw a rectangle that has that area:



(Area = 6 square units)

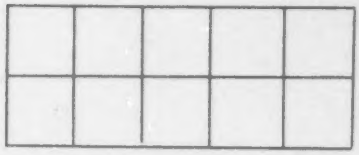
Then ask the students about the dimensions of the rectangle: What is the length? What is the width? Explain that each dimension is the sum of the linear units along that side.

Now, if the students know, for example, that the width is 2 units and the length is 3 units, let them multiply to find the product of 6. Point out that this is the same number as the number of square units within the rectangular region. Therefore, the area of a rectangle may also be found by multiplying the length and width of the region. Show them how this formula may be written ($A = l \times w$). Point out that using a formula such as this one is more efficient than counting square units. Areas of any-sized rectangles can be found using the formula whereas counting units, in some cases, would be next to impossible.

Contrast the area concept with the concept of perimeter. Have students find both the area and perimeter of given rectangular regions.

Examples: Find both the perimeter and the area of each of the following figures.

(a)



A= 10 square units
P= 14 units

(b)

A= 4 square units
P= 10 units

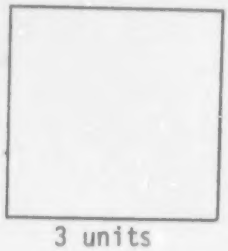
(c)

A= 12 square units
P= 14 units

(d)

A= 12 $cm^2$
P= 16 cm

(e)




A= 9 square units
P= 12 units

## 13.2 FINDING THE PERIMETER OF A RECTANGULAR REGION

### Common Errors

1. Makes computational errors.

2. Counts points along each side rather than units of length when given a rectangular region whose length and width have been marked off in units.

   Example: A student may indicate that the perimeter of the following figure is 20 units rather than 16 units:

3. Uses incorrect formulas to find the perimeter of a given rectangular region.

   Example: A student may find only half the perimeter because (s)he uses a formula such as: $P = L + W$.

4. Uses a correct formula but makes an error in the use of same.

   Example: A student may use a correct formula such as $2(l + w)$ in order to find the perimeter, but makes a mistake using the distributive property while attempting to find an answer:

$$\begin{aligned} & 2(l + w) \\ = \ & 2(l_2 + 8) \\ = \ & 2(12) + 8 \\ = \ & 24 + 8 \\ = \ & 32 \text{ units} \end{aligned}$$

5. Finds the area of a rectangular region rather than its perimeter.

Instructional Suggestions and Activities

Review computational algorithms. Instructional suggestions are given within computational skill descriptions.

It is important that the perimeter concept be developed through the use of visual aids. Substantial hands-on experiences with physical models of geometric shapes are needed. Pupils are not intuitively familiar with the concept of perimeter. They need experience measuring and counting units. Thus, show students that the perimeter of a rectangular region may be found by counting the total number of linear units along each side or by measuring the length of each side of the region and adding these lengths.

This error may be the result of a lack of understanding of the concept of perimeter. It is imperative that the concept of perimeter be fully developed prior to introducing any formulas. Formulas should not be introduced until pupils have a full understanding of the meaning of perimeter. As formulas are introduced, however, it may be necessary to review the properties of rectangular regions in order for students to make a connection between

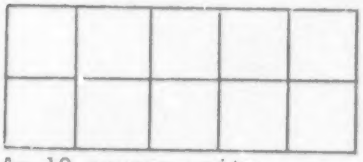
concepts and statements of formulas.

Review with students the types of operations that are indicated in the statement of various formulas. It may also be necessary to review correct order of operations. In the case of the use of the formula, $2(l + w)$, you need to review the use of the distributive property or point out that the length and width may be added and then that sum multiplied by two.

Contrast the perimeter concept with the area concept by having students find both the area and perimeter of given rectangular regions.

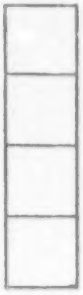
Examples: Find both the perimeter and the area of each of the following figures.

(a)



A= 10 square units
P= 14 units

(b)



A= 4 square units
P= 10 units

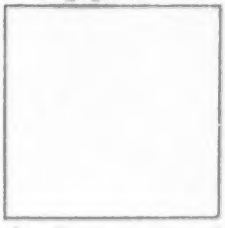
(c)

A= 12 square units
P= 14 units

(d)

A= 12 $cm^2$
P= 16 cm

(e)




A= 9 square units
P= 12 units

## 14.1 IDENTIFYING UNITS OF MEASURE THAT INDICATE LENGTH, MASS/WEIGHT, AND CAPACITY

### Common Errors

1. Confuses two (or more) units of measure with each other.

   Example: The student may indicate that the gram is a unit of capacity or a litre is a unit of mass.

### Instructional Suggestions and Activities

As students are involved in actual measurement experiences, emphasis should be placed upon matching the units of measure to the measurements of length, mass/weight, or capacity of various objects. Suggested measurement activities may be found in the Metric Education Plan for Virginia, Virginia Department of Education. Crossword puzzles such as the one given on the next page may provide appropriate practice for reviewing the metric units and prefixes.

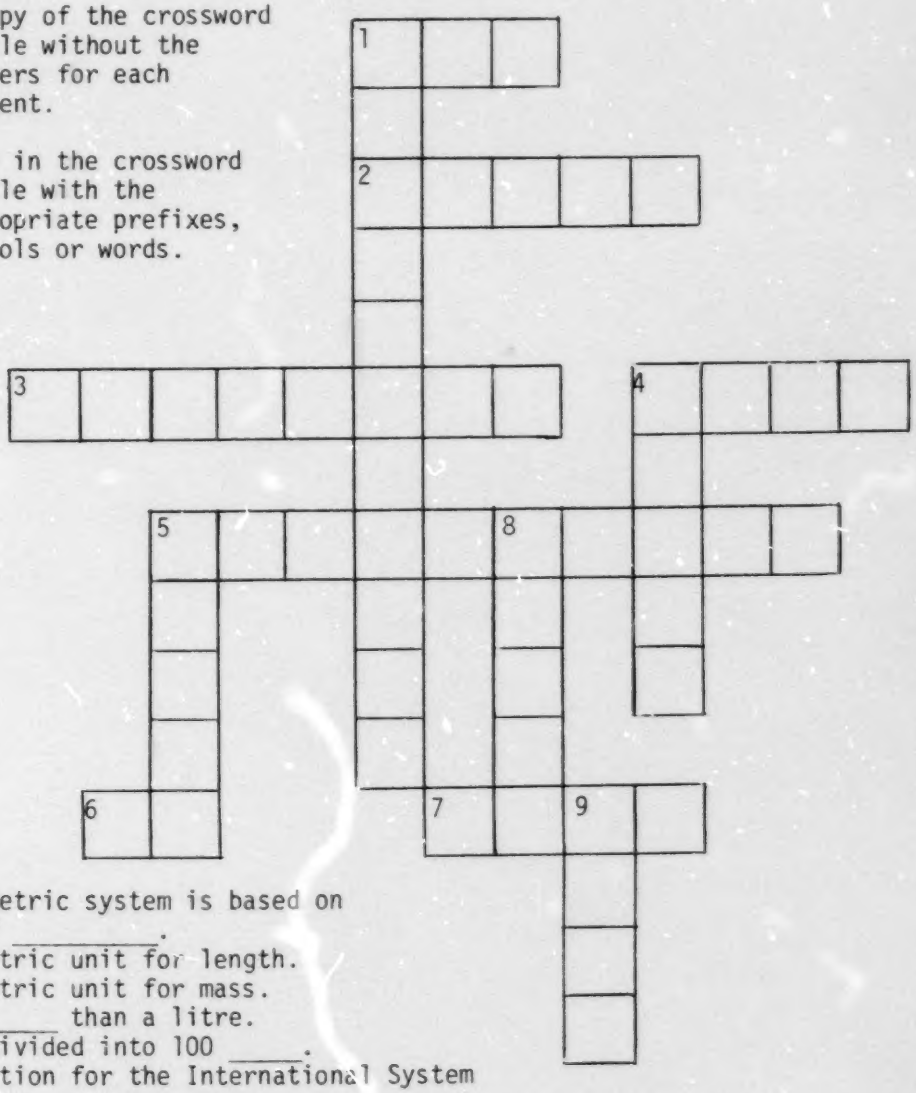
Note: During this time of transition to the metric system of measurement, it is necessary to teach both the English system and the metric system. However, the two systems should be taught independently of each other, with only informal comparisons made between the two.

METRIC CROSSWORD PUZZLE

Objective: To review the more commonly used metric units and the prefixes.

Equipment: A copy of the crossword puzzle without the answers for each student.

Procedure: Fill in the crossword puzzle with the appropriate prefixes, symbols or words.




ACROSS:

1. The entire metric system is based on multiples of _____.
2. The basic metric unit for length.
3. The basic metric unit for mass.
4. A quart is _____ than a litre.
5. A metre is divided into 100 _____.
6. The abbreviation for the International System of Units.
7. Metric prefix for 1000.

DOWN:

1. Degree Celsius is used for measuring _____.
4. The unit used in the metric system for capacity.
5. The prefix for 1/100.
8. The prefix for 1/1000.
9. The yard is _____ than a metre.

ACROSS:
1. Ten
2. Metre
3. Kilogram
4. Less
5. Centimetres
6. SI
7. Kilo

DOWN:
1. Temperature
4. Litre
5. Centi
8. Milli
9. Less

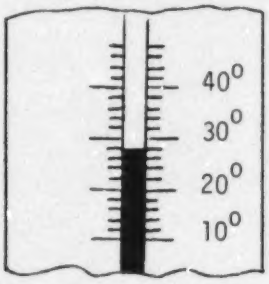
## 14.2 WRITING INDICATED TEMPERATURES TO THE NEAREST DEGREE

### Common Errors

1. Incorrectly determines the point at which the shaded area representing the filament in a thermometer terminates on the scale.

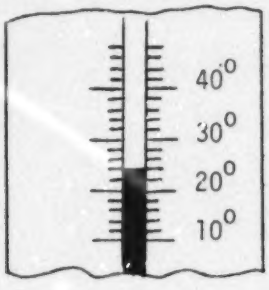
2. Reads the degree markings on a scale in reverse order.

   Example: A temperature of $28^{o}$ F may be read as one of $32^{o}$ F.




3. Incorrectly determines the number of degrees each mark on a thermometer represents.

   Example: A temperature of $24^{o}$ C may be read as one of $22^{o}$ C (when actually each mark represents a $2^{o}$ change in temperature).




### Instructional Suggestions and Activities

Have students place a straight edge along the top of the filament. Then have them read across to the scale in order to find the number of degrees of temperature.

Review reading scales that have been placed in a vertical position. Point out that degree markings represent higher temperatures as one moves up the scale. That is, the $32^{o}$ mark is above the $30^{o}$ mark on the scale. Several practice activities should be provided in which students have to read various scales. You may want to begin these practice sessions by having them read scales on number lines.

Provide practice exercises where students must read scales and number lines that have been marked in various types of intervals. It may be helpful for students to read temperatures from thermometers that have been placed in the classroom, in the hall, outside the building, in ice water or in hot water.

## 14.3 DETERMINING ELAPSED TIME

### Common Errors

1. Counts the numbers at the hour designations on the clock face rather than counting hour intervals.

   Example: A student may indicate that 3 hours has elapsed between 2:00 p.m. and 4:00 p.m. because (s)he has counted the numbers 2, 3, and 4 on the clock face.

2. Can only count elapsed time from a point on the hour.

   Example: The student may not be able to indicate that 2 hours have passed from 2:30 p.m. to 4:30 p.m. but could do so if (s)he had been asked how much time had elapsed from 2:00 p.m. to 4:00 p.m.

3. Confuses a.m. and p.m. time designations.

### Instructional Suggestions and Activities

Students need practice finding elapsed time, using any one of several methods. However, prior to this practice, they need to review the difference between a.m. and p.m. time designations as well as how to count time intervals using a clock face. Practice exercises should also include finding elapsed time from a point other than on the hour.

Elapsed time may be found several ways.

1. If both times are either a.m. or p.m., you may, for example, indicate the elapsed time from 1:30 p.m. to 3:00 p.m. using the following procedures:

   a. 3:00 p.m. is 3 hours 0 minutes after noon
      1:30 p.m. is 1 hour 30 minutes after noon

      So,

3 hours 0 minutes = 2 hours 60 minutes
- 1 hour 30 minutes = 1 hour 30 minutes
1 hour 30 minutes

∴ The elapsed time from 1:30 to 3:00 p.m. is 1 hour 30 minutes or 1½ hours.

b. Counting from 1:30 p.m. to 3:00 p.m.:

1:30 p.m. - 2:30 p.m. = 1 hour
2:30 p.m. - 3:00 p.m. = ½ hour

∴ 1 hour + ½ hour = 1½ hours or 1 hour 30 minutes

2. If one time is a.m. and the other time is p.m., you may, for example, indicate the elapsed time from 10:00 a.m. to 2:30 p.m. using the following procedures:

a. Use 12:00 Noon as a reference point since it serves as a "dividing point" for a.m. and p.m. time designations.

12:00 Noon is 12 hours 0 minutes after midnight

10:00 a.m. is 10 hours 0 minutes after midnight

So,

12 hours 0 minutes
- 10 hours 0 minutes
2 hours 0 minutes (elapsed time from 10:00 a.m. to 12:00 Noon)

Also, 2:30 p.m. is 2 hours 30 minutes after Noon.

So,

2 hours 30 minutes
+ 2 hours 0 minutes
4 hours 30 minutes

∴ The elapsed time from 10:00 a.m. to 2:30 p.m. is 4 hours 30 minutes or 4½ hours.

b. Counting from 10:00 a.m. to 2:30 p.m.:

10:00 a.m. - 11:00 a.m. = 1 hour
11:00 a.m. - 12:00 Noon = 1 hour
12:00 Noon - 1:00 p.m. = 1 hour
1:00 p.m. - 2:00 p.m. = 1 hour
2:00 p.m. - 2:30 p.m. = ½ hour

1 hr. + 1 hr. + 1 hr. + 1 hr. + ½ hr. = 4½ hours or 4 hours 30 minutes

Note: You may want to use train, bus, or plane schedules to make practice more appealing to students.

## 15.1 DETERMINING CORRECT CHANGE

### Common Errors

1. Makes computational errors.

2. Counts incorrectly when determining change by adding onto the amount of purchase.

3. Writes the amount of change using incorrect dollar and cent notation.

   Example: A student may write $3.05 when (s)he actually means that the change is $3.50.

### Instructional Suggestions and Activities

Review computational algorithms with students. See the appropriate skill descriptions for suggested instructional activities.

Students need practice counting change. It may be beneficial to put students in role-playing situations where they use "coins" to figure the amount of change. They also need to count change to each other in order to become more familiar with coin value. An appropriate role-playing situation can be developed from a problem such as the following one:

> Mary goes to Best Burger to buy a cheeseburger and medium coke. The cheeseburger costs $0.42 and the coke costs $0.30. Mary gives Sam, the cashier, a five-dollar bill. Sam takes the money and counts out the change to Mary. How much change does he give her?

Students need to practice writing amounts of money in correct dollar and cent notation. It may be appropriate to call out amounts of money and have the students write the dollar and cent notation for each amount on their papers.

## 15.2 COMPUTING HOURLY WAGES

Common Errors

1. Makes computational errors.

2. Uses incorrect operation(s) to compute wages.

   Example: A student may divide the hourly rate of pay by the number of hours worked.

Instructional Suggestions and Activities

Review the multiplication algorithm. See the multiplication skill descriptions for suggested instructional activities.

Terminology may be getting in the student's way. Hence, a review of the meaning of such terms as wages and hourly rate of pay may be necessary.

Students may also need to practice finding wages. Using actual job experiences of various students in your classroom and sample time cards provide excellent sources of exercises.

15.3 COMPUTING NET EARNINGS

Common Errors

1. Makes computational errors.

2. Uses incorrect operation(s) to compute the net earnings.

   Example: A state policeman earns a salary of $1200 a month. What is his take-home pay each month after the following deductions are made: Federal Income Tax - $161.40; State Income Tax - $51.50; Social Security - $73.15?

   A student may attempt to solve this problem in the following manner:

$$\begin{array}{r} \$1200.00 \\ 161.40 \\ 51.50 \\ +\ 73.15 \\ \hline \$1486.05 \end{array}$$

3. Confuses technical terms such as gross earnings, net earnings, take-home pay, etc.

Instructional Suggestions and Activities

Review computational algorithms as needed. See the appropriate computational skill descriptions for suggested instructional activities.

Use sample check stubs, if possible, and go over with students how the net income is figured from the gross income. Using job experiences of students as examples provides much motivation.

NOTE: You may want to give students the suggestion that their net income is approximately 2/3 of their gross income.

Work with the students on terminology. Many times students can find correct answers if the vocabulary does not get in the way. We use terms like "take-home pay" synonymously with net earnings or net income. This certainly can be confusing to students. Use plenty of examples during the review time. Memorizing dictionary definitions is not of any benefit.

## 15.4 WRITING A CHECK

Common Errors

1. Fails to include date on the check.

2. Places the incorrect information in the blanks on the check.

   Example: A student may put his own name in the "Pay to the order of" blank.

3. Writes the amount of the check incorrectly.

   Example: The student may write the amount of the check in the dollar and cent notation incorrectly or he may write the word name for the amount incorrectly.

4. Writes the amount in words different from the amount in dollar and cent notation.

Jane Wilson
Route 1
Flatville, VA 00346 September 30, 19 78

Pay to the
Order of Lois Watkins $ 5.05

Five and 50/100----------Dollars

For Cosmetics Jane Wilson

5. Signs the check incorrectly.

Examples: a. A person may print his name in the signature blank on a check rather than sign it.

b. The name written as a signature on a check may not totally match the name printed in the upper left-hand corner of the check. That is, a check may be signed Butch Worth but the name E. L. Worth, Jr., is printed on the check.

Instructional Suggestions and Activities

Emphasize the importance of putting dates on documents. It makes record-keeping so much more accurate and efficient.

Go over with students the type of information that goes in each blank on a check. Have the students fill out many sample checks.

The following activity provides practice as well as motivation for writing checks:

> Give each student a job with a reasonable income. Have each one open a checking account. Have sample checks available for distribution. Upon having students deposit money in their accounts, give them catalogs from which to make purchases. These purchases are to be paid for by check.

Look closely at sample checks students have written to determine whether the mistake is with the dollar and cent notation or with the word name for the amount of the check; then review accordingly. It is also possible that a student does not understand how to write "cents" as a fractional part of a dollar, so this notation may also need review.

Provide an exercise where the students must match the amount of the check in words to that amount in dollar and cent notation.

Stress the importance of not printing your name when you sign a check. It is too easily forged. Your signature on a check must match the signature on a card that you sign when your account is opened. Banks will not accept checks that have not been properly signed. Also, your check may be placed in someone else's account if your name in the signature blank is different from that printed on your check.

15.5 COMPLETING A CHECK STUB

Common Errors

1. Makes computational errors.

2. Places entries under wrong headings.

   A student may place entries under incorrect headings because (s)he has no understanding of the terminology (Example: Balance Forwarded) or of abbreviations (Example: Bal. For'd.) used on stubs.

3. Uses incorrect operation(s) in order to complete the stub.

   Example: A student may add the amount of the check to the balance forwarded rather than subtract the amount from the balance.

Instructional Suggestions and Activities

Review computational algorithms as needed. See the appropriate computational description for suggested instructional activities. Stress the importance of a correct balance on hand.

Review with students the meaning of each term and abbreviation on a check stub. Remember that there are various types of stubs and students need to be familiar with each.

It is important that students actually fill out check stubs from a given set of information. In order to provide motivation, students may need to be involved in role-playing situations where they have their own checking accounts, write their own checks, and fill out the stubs. As the students are involved in such activities, stress the kinds of operations that must be used to complete the stubs correctly.

## 15.6 DETERMINING AND COMPARING UNIT PRICES OF SIMILAR PACKAGED GOODS

Common Errors

1. Makes computational errors.

2. Uses incorrect operation(s) to determine unit prices.

3. Compares numerical values incorrectly.

   Example: A student may indicate that a can of Brand X milk at 7 1/4¢ per ounce is cheaper than a can of Brand Y milk at 7 1/10¢ per ounce.

4. Compares unlike units.

   Example: A student may compare the price per ounce of a 24-oz. can of Star Baked Beans to the price per quart of a 2-quart can of Lance Baked Beans.

5. Assumes that the more you buy of an item, the cheaper it is and, thus, does not do any calculating or estimating to arrive at the answer.

   Example. A student may assume that the unit price of a 20-oz. can of Made-Rite Tomato Juice costing 85¢ is less than the unit price of a 12-oz. can of Keep-Cool Tomato Juice costing 48¢ just because the can of Made-Rite Tomato Juice is larger.

Instructional Suggestions and Activities

Review computational algorithms as needed. See the appropriate computational skill descriptions for suggested instructional activities.

Students may not understand what is meant by the term "unit price". Show them examples of how unit prices are found. Then have them practice finding unit prices of items. Use grocery ads or prices of items obtained from local supermarkets as a part of the practice exercises. An activity that provides practice as well as motivation is as follows: Give the students a grocery list and tell them they must "buy" the items using the smallest amount of money possible. (You may want to use items that appear in supermarket ads so you can check their work.)

Review ordering numbers. See the appropriate skill description for suggested instructional activities.

Emphasize the importance of first looking at the unit before performing any operation to compare unit prices. If the units are not the same, they must be changed to the same unit prior to unit price comparison. It is usually easier for students to change larger units to smaller equivalent units (2 qt. = 64 oz.).

Show students examples where they cannot assume that the more you buy, the cheaper it is. You may want to explain that you cannot always make this assumption because of product quality and name.

## 15.7 COMPUTING THE AMOUNT OF DISCOUNT

Common Errors

1. Makes computational errors.

2. Uses incorrect operation(s) to compute the amount of discount.

   Example: The student may divide the regular price of an item by the rate of discount to find the amount of discount rather than multiply the price by the rate.

3. Expresses percent incorrectly as a decimal or fraction.

4. Finds the sale price of an item rather than the amount of discount.

Instructional Suggestions and Activities

Review computational algorithms as needed. See the appropriate computational skill descriptions for suggested instructional activities.

Emphasize that the amount of discount must be smaller than the regular price of an item. Also, review with students the notion that since the rate of discount is a percent, then finding the amount of discount is the same procedure as finding the percent of a number. Thus, the amount of discount is found by multiplying the rate by the regular price of an item.

Review changing a percent to a decimal or a fraction. See the appropriate skill descriptions for suggested instructional activities.

If a student indicates the sale price of an item rather than the amount of discount, (s)he is merely going one step too far. (S)he knows how to find the amount of discount otherwise (s)he would not have been able to find the sale price. Thus, emphasis should be placed on reading the problems carefully in order to find the requested information.

APPENDIX A

STATE ADOPTED COMPETENCIES FOR MATHEMATICS

| Student Competencies | Performance Indicators |
|---|---|
| The student will be able to: | |
| 1. Read and write numerals correctly. | 1.2 Given numerals naming whole numbers less than ten thousand, the student will write the word name for the number. |
| 2. Compare numerical values of whole numbers. | 2.1 Given a set of whole numbers, named by numerals of not more than four digits, the student will arrange the numerals in order from smallest numerical value to largest. |
| 3. Add, subtract, multiply, and divide whole numbers. | 3.1 Given two, three, or four whole numbers named by numerals of not more than four digits, the student will find the sum. |
| | 3.2 Given two whole numbers named by numerals of not more than three digits, the student will find the difference. |
| | 3.3 Given two whole numbers named by numerals of not more than three digits, the student will find the product. |
| | 3.4 Given a dividend named by a numeral of no more than four digits and a divisor named by a numeral of no more than two digits, the student will find the quotient. |
| 4. Add, subtract, multiply, and divide decimal fractions. | 4.1 Given two decimal fractions named by numerals of no more than four digits and having no more than three places to the right of the decimal point, the student will find the sum or difference. |

| Student Competencies | Performance Indicators |
|---|---|
| The student will be able to: | |
| | 4.2 Given two decimal fractions, named by numerals of three digits with the decimal point in any position, and the digits of their product, the student will properly place the missing decimal in the product. |
| | 4.3 Given a dividend of no more than four digits and no more than three decimal places, a divisor of no more than two digits and no more than one decimal place, and the digits of the quotient, the student will properly place the decimal point in the quotient. |
| 5. Multiply simple fractions. | 5.1 Given two simple fractions which have denominators of 1, 2, 3, 4, or 5, the student will find the product. |
| 6. Express percents as equivalent decimal fractions. | 6.1 Given a percent, the student will write the equivalent decimal fraction. |
| 7. Express fractions as equivalent decimal numbers. | 7.1 Given a common fraction which has a single digit denominator, the student will write the equivalent decimal fraction. |
| 8. Find a given percent of a number. | 8.1 Given a number and a percent (from 1-100 inclusive) the student will find the percentage. |
| 9. Read and draw conclusions from information contained in simple bar, broken-line and circle graphs. | 9.1 Given a bar or broken-line graph, the student will estimate and compare quantitative information. |
| | 9.2 Given a circle graph in which each sector is labeled as a percent of the whole, the student will compare quantitative information. |

| Student Competencies | Performance Indicators |
|---|---|
| The student will be able to: | |
| | 9.3 Given a bar, broken-line, or circle graph and a statement of inference, the student will state whether the inference is true or false based on information on the graph. (The inference will be a simple statement relative to comparisons of size or frequency and trends of increase, decrease, or constancy.) |
| 10. Determine distance based on information from a given map. | 10.1 Given a map and scale, the student will indicate the route of least mileage between two locations. |
| 11. Read tables to determine taxes, interest, or insurance premiums. | 11.1 Given a specified situation and a federal or state income tax table, the student will find the correct amount of tax. |
| | 11.2 Given the cost of an item(s) and a sales tax table, the student will find the correct amount of sales tax. |
| 12. Identify objects and/or lines positioned in such a manner that the concept of parallelism is represented. And, identify certain parts - center, radius, and diameter - of a circle. | 12.1 Given a set of figures, the student will identify those which best represent the concept of parallelism. |
| | 12.2 Given an appropriate drawing of a circle, the student will identify the center, a radius, and a diameter. |
| 13. Determine the perimeter and area of rectangular regions. | 13.1 Given the dimensions of a rectangular region, the student will find the area of the region. |
| | 13.2 Given the lengths of the sides of a rectangular region, the student will find the perimeter of the region. |
| 14. Identify units of measure for length, mass (weight), and capacity; read a thermometer; and determine elapsed time. (Maximum 12-hour time interval.) | 14.1 Given a list of units of measure, the student will identify those indicating length (metre, centimetre, kilometre; foot, inch, mile) mass/weight (kilogram, gram; pound) and capacity (litre, millilitre; pint, quart, gallon), or vice versa. |

| Student Competencies | Performance Indicators |
| --- | --- |
| The student will be able to: | |
| | 14.2 Given a drawing of either a Celsius or Fahrenheit thermometer, the student will write the indicated temperature to the nearest degree. |
| | 14.3 Given two times (to the nearest ½-hour designation) within a 12-hour time interval, the student will determine the elapsed time. |
| 15. Solve practical problems dealing with banking, consumer purchases, and personal earnings. | 15.1 Given an amount of money, $1, $5, or $10, and the total amount of purchase, the student will determine the correct change. |
| | 15.2 Given a specified number of hours worked and an hourly rate of pay, the student will compute the wages. |
| | 15.3 Given the gross earnings and the amounts of deduction for social security and federal and state taxes, the student will compute the net earnings. |
| | 15.4 Given the appropriate information, the student will write a check. |
| | 15.5 Given the appropriate information, the student will complete a check stub. |
| | 15.6 Given the prices of similar packaged goods the student will determine and compare unit prices. |
| | 15.7 Given the regular price of an item and a rate of discount, the student will compute the amount of discount. |

## APPENDIX B

### Vocabulary List

These words, terms, and abbreviations are representative of the mathematical language which appears in problems suggested by the minimum competencies. It is not intended that students' know dictionary meanings of these terms but that they understand them in the context of problem solving situations. For your convenience, the list has been broken down into four general categories. This should not be interpreted to mean that a given word or term is only associated with that particular category.

#### Graphs, Charts, and Tables

adjusted gross income
at least
bar graph
broken-line graph
circle graph
constant
decrease
deduction
distance
greater than
greatest
half as much
income tax
increase
least
less than
mileage
more than
most
route
sales tax
simple interest
taxable income
twice as much

Geometry and Measurement

a. m.
area
capacity
Celsius (C)
centimetre
circle
diameter
elapsed time
Fahrenheit (F)
foot
gallon
gram
inch
kilogram
kilometre
length
line segment
litre
mass
metre
mile
millilitre
parallel
perimeter
perpendicular
pint
p. m.
pound
quart
radius
rectangle
square units (EX: $cm^2$ and sq. in.)
thermometer
weight

Numeration and Computation

common fraction
decimal fraction
difference
equal
largest
numerical value
percent
percentage
product
quotient
rename
smallest
sum

Practical Applications

amount of deposit
amount of discount
amount of purchase
balance
balance brought forward
deduction
deposit
gross earnings
net earnings
unit price
wages

APPENDIX C

A BRIEF DESCRIPTION
of
COMMONWEALTH OF VIRGINIA
GRADUATION COMPETENCY TEST - MATHEMATICS*

The Commonwealth of Virginia Graduation Competency Test - Mathematics was developed by Scholastic Testing Service and is designed to measure those minimum competencies in mathematics adopted by the Board of Education, June, 1978. Due to the limited amount of time available for developing the test which was designed especially for Virginia, an instructional manual is not presently available from Scholastic Testing Service. The Mathematics Service of the Department of Education, in an effort to fill this void, is in the process of developing a manual to assist educators in the overall competency program as it relates to mathematics. Since this publication will not be available until later in the year, this brief description is being prepared to give general information on the Mathematics Test.

General Description

The test consists of 99 items, three items per performance indicator, with one exception. The performance indicator "Given numerals naming whole numbers less than ten million, the student will read the numerals" is not included. The test is multiple choice with four choices per item. Each item has one and only one correct choice. Choices such as "all of the above" or "none of the above" are not included as options. Most examples are presented in vertical form, however, a few are listed horizontally. Approximately one-half of the test is made up of problems with little reading required except numbers. The rest of the test includes word problems with some of these containing visual presentations to use as a basis for solving the problem. The solving of these problems would require reading, interpretation, and computation.

General Content and Sample Test Item

What follows is a general listing of content included on the test and also sample test items. For more detailed information on the content, consult the minimum competencies and performance indicators.

*This is a copy of materials sent to Key Instructional Leaders October 3, 1978.

General Content Area

1. Write word names for numbers.----------3 test items

   Sample Test Item

   The word name for 3,062 is

   a) three hundred sixty-two
   b) thirty-six hundred two
   c) three thousand sixty-two
   d) three thousand six hundred two

2. Compare numerical values.----------3 test items

   Sample Test Item

   Which set of numbers goes from the smallest to the largest?

   a) {110, 249, 72, 300, 515}
   b) {3, 7, 9, 5, 8}
   c) {39, 27, 14, 11, 1}
   d) {7, 16, 21, 27, 41}

3. Computation with whole numbers (addition, subtraction, multiplication, and division)----------12 test items

   Sample Test Item

   Divide: $46\overline{)552}$

   a) 11
   b) 12
   c) 13
   d) 14

4. Computation with decimals (addition, subtraction, multiplication, and division)----------9 test items

   Sample Test Item

   Add: 12.7 + 0.36 = ☐

   a) .487
   b) 1.63
   c) 13.06
   d) 12.43

5. Multiply with simple fractions.----------3 test items

Sample Test Item

Multiply: 1/4 x 2/3 = ☐

a) 1/6
b) 1/2
c) 3/8
d) 3/7

6. Renaming fractions (common and decimal) and computation with percents.----------9 test items

Sample Test Item

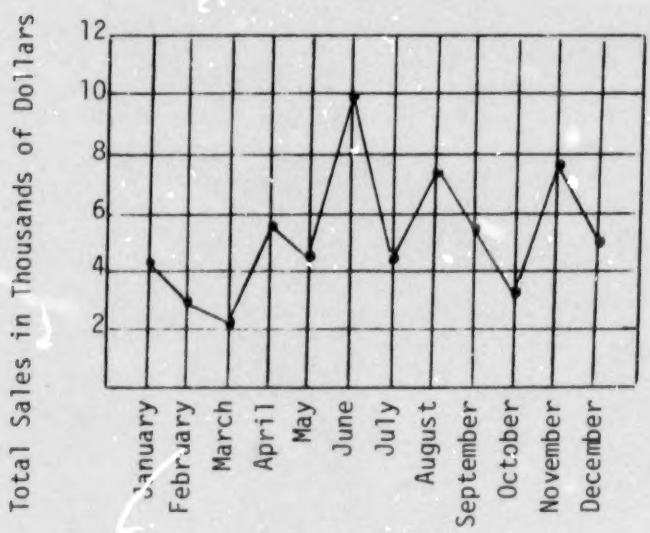
15% of 80 is

a) 120
b) 12
c) 5
d) 15

7. Reading and interpreting graphs, tables, and charts.----------18 test items

Sample Test Item

Manufacturing Company's Monthly Sales for 1976



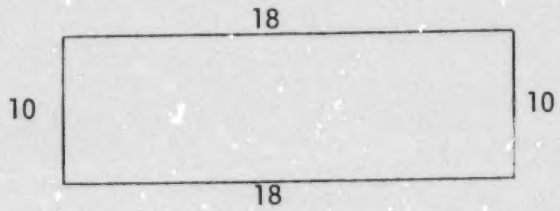
According to the graph, which statement is true?

a) Company sales decreased from March to May
b) Company sales increased from October to November
c) Company sales remained constant from January to March
d) Company sales were the same amount for April and November

8. Geometric concepts and relations.----------12 test items

Sample Test Item

What is the perimeter of this figure?




a) 28
b) 180
c) 36
d) 56

9. Identify units of measure and determine elapsed time.----------9 test items

Sample Test Items

Which measures length?

a) kilogram
b) metre
c) litre
d) milligram

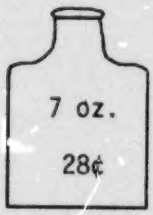
The children left school at 11:00 a.m. and returned at 2:30 p.m. How long were they away from school?

a) 2 1/2 hours
b) 5 1/2 hours
c) 4 1/2 hours
d) 3 1/2 hours

10. Solve problems with practical applications.----------21 test items

Sample Test Item

Which bottle of juice is the cheapest per ounce?

a) 


b)


c)


d)


Using the check record below, what is the new balance after checks #672 and #673 are written?

PLEASE BE SURE TO DEDUCT ANY PER CHECK CHARGES OR SERVICE CHARGES THAT MAY APPLY TO YOUR ACCOUNT

| CHECK NO | DATE | CHECKS ISSUED TO OR DESCRIPTION OF DEPOSIT | (-) AMOUNT OF CHECK | | ✓ T | (-) CHECK FEE (IF ANY) | (+) AMOUNT OF DEPOSIT | | BALANCE | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | 120 | 00 |
| 672 | 8/30 | To: Sears<br>For: radio | 26 | 00 | | | | | | |
| 673 | 9/6 | To: Auto Supply Co.<br>For: batteey | 32 | 00 | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |
| | | | | | | | | | | |

a) $ 88.00
b) $ 94.00
c) $ 62.00
d) $178.00

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